Impactors and Particle Size Distribution (2)

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Particle size statistics
Aerosol measurement and size distribution

1. Concentration
Particle mass, surface area or number per unit volume

2. Size distribution
Concentration versus particle size

Aerosol particle sizing

Particles are assigned to bins according to particle diameter
Conversion of a discrete particle size distribution to a continuous distribution

Number concentration is proportional to the height of each bar.

Continuous Distribution
Number concentration is proportional to the area of each bar.
Continuous graph

- **Y-axis**
  - Differential particle concentration (or number)
    - Particle number normalized by range of particle diameter of the interval (or bin)
    - \( \frac{dn}{dd} \)
  - **X-axis**
    - Particle diameter range of the interval
      - \( dd \)
  - Integrated area under the curve = total # of particle
    \[ \int \frac{dn}{dd} dd = n \]
Properties of particle size distribution

- Asymmetrical (or skewed) distribution
  - Long tail to the right
    - Large number (fraction) of small particles
    - Small number (fraction) of large particles
  - Large range of particle size
    - Several orders of magnitude in particle diameter
  - No negative particle size
- Mode < Median < Mean
- Geometric mean ($d_g$ or GM)

$$\log d_g = (\sum n_i \log d_i) / N \Rightarrow d_g = \exp\left\{ (\sum n_i \log d_i) / N \right\}$$

Arithmetic mean and GM

- Arithmetic mean
  $$d_a = (\sum n_i d_i) / N = \text{(summation of all areas of the bars)} / \text{(total number of particles)}$$

- Geometric mean
  $$\log d_g = (\sum n_i \log d_i) / N \Rightarrow d_g = \exp\left\{ (\sum n_i \log d_i) / N \right\}$$
The smooth continuous distribution is obtained by joining the bin mid-points.

Asymmetric (skewed) distribution

Mode

Arithmetic mean

Geometric mean (Median)

Continuous Distribution

Note that when plotted as \( \frac{dN}{d(d)} \), the modal and count median diameters of a lognormal distribution are different.
Normal distribution

\[ df = \frac{n}{\sigma \sqrt{2\pi}} e^{-\frac{(d_p - \bar{d}_p)^2}{2\sigma^2}} d\bar{d}_p \]

- \( \bar{d}_p \): arithmetic mean particle diameter
- \( \sigma \): standard deviation
- \( d\bar{d}_p \): particle diameter interval
- \( df \): frequency of occurrence of particles of diameter \( dp \)
Log transformation of continuous graph

- **Y-axis**
  - Differential particle concentration (or number)
    - Particle number normalized by range of log-transformed particle diameter of the interval (or bin)
    - $dn/d\log(d)$
  
- **X-axis**
  - Range of log-transformed particle diameter of the interval
    - $d\log(d)$

- Integrated area under the curve=total # of particle
  - $\int \{dn/d\log(d)\} d\log(d)$

Lognormal distribution with arithmetic scale

- Count median diameter (CMD) = GM
  - Note that when plotted as $dn/d\log(d)$, the modal and count median diameters of a lognormal distribution are different
  - Distribution skewed to smaller particle diameters

- Normalized distribution (n=1)
  - Area under curve = 1

- Modal Diameter

- Lognormal Size Distribution
Mathematical function of lognormal distribution

\[ df = \frac{n}{\sqrt{2\pi \text{Log}(\sigma_g)}} e^{-\frac{\left(\text{Log}(d_p) - \text{Log}(\text{CMD})\right)^2}{2\text{Log}(\sigma_g)^2}} \text{dLog}(d_p) \]

\[ df = \frac{n}{\sigma \sqrt{2\pi}} e^{-\frac{(d_p - \bar{d}_p)^2}{2\sigma^2}} \text{dd}_p \]

Two ways of log-transformed graph

- Transform the original particle size data using logarithm, and then plot them on normal arithmetic scale of the graph
  - To calculate all statistics mathematically
  - Exponentiate log-transformed statistics
- Transform x-axis scale of the graph, and then plot the original particle size data on it
  - Do not transform the data
  - Only change the scale of the graph
Log normal distribution (first approach)

- Normalized distribution (n=1)
  - Area under curve = 1
- Mean = Mode = Median
- 68% of all particles are between $d \pm \sigma$
- Distribution characterized by $n$, $\bar{d}$, and $\sigma$

Lognormal Size Distribution

- Count Median Diameter (CMD)
- 16% of all particles are less than CMD/$\sigma$
- 50% of all particles are less than CMD
- 68% of all particles are between CMD/$\sigma$ and CMD $\times \sigma$

Lognormal distribution with log scale

- Normalized distribution (n=1)
  - Area under curve = 1
- Count Median Diameter (CMD)
- Lognormal Size Distribution
- 68% of all particles are between CMD/$\sigma$ and CMD $\times \sigma$
- 84% of all particles are less than CMD$\times\sigma$
- Distribution characterized by $n$, CMD, and $\sigma$

- 16% of all particles are less than CMD/$\sigma$
- 50% of all particles are less than CMD
- 68% of all particles are between CMD/$\sigma$ and CMD $\times \sigma$
- 84% of all particles are less than CMD$\times\sigma$
Cumulative size distribution

Probability scale of lognormal distribution

- Percent of particles less than a given particle diameter
  - $<\text{CMD}/(2\sigma_g)$ - 5%
  - $<\text{CMD}/\sigma_g$ - 16%
  - $<\text{CMD}$ (median) - 50%
  - $<\text{CMD}^*\sigma_g$ - 84%
  - $<\text{CMD}^*(2\sigma_g)$ - 95%
- Any pattern??
  - Symmetry of probability
Lognormal distribution with log scale

Normalized distribution \( n=1 \)
Area under curve = 1

Count Median Diameter (CMD)

50% of all particles are less than CMD

68% of all particles are between CMD / \( \sigma \) and CMD * \( \sigma \)

84% of all particles are less than CMD * \( \sigma \)

Distribution characterized by \( n \), CMD and \( \sigma \)

Cumulative plot to log-probability plot

Switch and change to probability scale
Log-probability paper

Log-probability plot of lognormal distribution

\[ 1/2 = \sigma_g = 2/3 \]
Count median diameter and $\sigma_g$

- CMD (count median diameter)
  - 50% of all particles are less than CMD
- Geometric standard deviation
  - $\text{CMD} \times \sigma_g / \text{CMD}$ or
  - $\text{CMD} / (\text{CMD} / \sigma_g)$

![CMD, SMD, and MMD](image)
Log-probability plots for count, mass, and surface area lognormal distribution

Plots are parallel - $\sigma_g$ (given by gradient) is the same for each weighting

Cascade impactor data reduction

<table>
<thead>
<tr>
<th>Stage #</th>
<th>Size Range ($\mu$m)</th>
<th>$d_{50}$ ($\mu$m)</th>
<th>Initial Mass (mg)</th>
<th>Final Mass (mg)</th>
<th>Net Mass (mg)</th>
<th>Mass Fraction (%)</th>
<th>Cumm. Mass Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&gt;9.0</td>
<td>9.0</td>
<td>850.5</td>
<td>850.6</td>
<td>0.1</td>
<td>0.6</td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>4.0-9.0</td>
<td>4.0</td>
<td>842.3</td>
<td>844.1</td>
<td>1.8</td>
<td>11.0</td>
<td>99.4</td>
</tr>
<tr>
<td>3</td>
<td>2.2-4.0</td>
<td>2.2</td>
<td>855.8</td>
<td>861.0</td>
<td>5.2</td>
<td>31.7</td>
<td>88.4</td>
</tr>
<tr>
<td>4</td>
<td>1.2-2.2</td>
<td>1.2</td>
<td>847.4</td>
<td>853.6</td>
<td>6.2</td>
<td>37.8</td>
<td>56.7</td>
</tr>
<tr>
<td>5</td>
<td>0.7-1.2</td>
<td>0.70</td>
<td>852.6</td>
<td>855.1</td>
<td>2.5</td>
<td>15.2</td>
<td>18.9</td>
</tr>
<tr>
<td>Back filter</td>
<td>0-0.7</td>
<td>0</td>
<td>78.7</td>
<td>79.3</td>
<td>0.6</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>
An example using count data

Data

<table>
<thead>
<tr>
<th>Size Range * (µm)</th>
<th>Count</th>
<th>Fraction/µm</th>
<th>Percent</th>
<th>Cumulative Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td>104</td>
<td>0.026</td>
<td>10.4</td>
<td>10.4</td>
</tr>
<tr>
<td>4–6</td>
<td>160</td>
<td>0.080</td>
<td>16.0</td>
<td>26.4</td>
</tr>
<tr>
<td>6–8</td>
<td>161</td>
<td>0.0805</td>
<td>16.1</td>
<td>42.5</td>
</tr>
<tr>
<td>8–9</td>
<td>75</td>
<td>0.075</td>
<td>7.5</td>
<td>50.0</td>
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<tr>
<td>9–10</td>
<td>67</td>
<td>0.067</td>
<td>6.7</td>
<td>46.7</td>
</tr>
<tr>
<td>10–14</td>
<td>186</td>
<td>0.0465</td>
<td>18.6</td>
<td>75.3</td>
</tr>
<tr>
<td>14–16</td>
<td>61</td>
<td>0.0305</td>
<td>6.1</td>
<td>81.4</td>
</tr>
<tr>
<td>16–20</td>
<td>79</td>
<td>0.0197</td>
<td>7.9</td>
<td>89.3</td>
</tr>
<tr>
<td>20–35</td>
<td>90</td>
<td>0.0060</td>
<td>9.0</td>
<td>98.3</td>
</tr>
<tr>
<td>35–50</td>
<td>17</td>
<td>0.0011</td>
<td>1.7</td>
<td>100.0</td>
</tr>
<tr>
<td>&gt;50</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Total</td>
<td>1000</td>
<td></td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

*Intervals are equal to or greater than the lower limit and less than the upper limit.

Plotting upper size range vs. fraction/µm

Log scale of particle diameter (lognormal distribution)
Take-home practice

<table>
<thead>
<tr>
<th>Stage cut-point (d) (µm)</th>
<th>Mass concentration (mg/m^3)</th>
<th>Cumulative mass concentration greater than diameter d (mg/m^3)</th>
<th>%Cumulative mass concentration greater than diameter</th>
<th>%Cumulative mass concentration less than diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.4209</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.62</td>
<td>1.3330</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.16</td>
<td>3.3770</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.78</td>
<td>6.4771</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9.5248</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>12.7579</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.285</td>
<td>8.1861</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.158</td>
<td>5.2389</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>2.9853</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Fill out the blank columns.
2. Find out MMD (Mass Median Diameter) and Geometric standard deviation (GSD).