Impactors and Particle Size Distribution (2)

National Institute for Occupational Safety and Health
Division of Respiratory Disease Studies
Field Studies Branch

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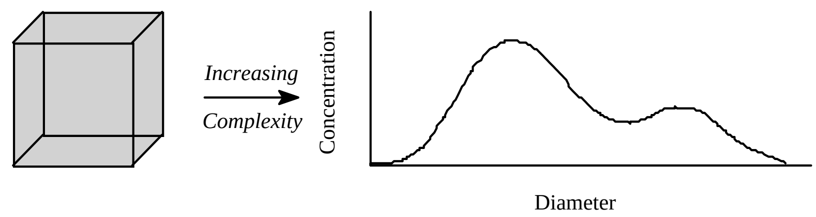
Particle size statistics

Aerosol measurement and size distribution



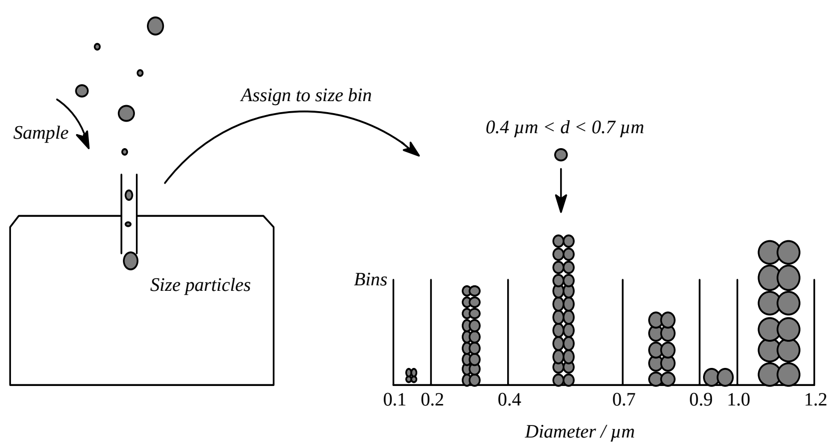
1. Concentration

Particle mass, surface area or number per unit volume

2. Size distribution

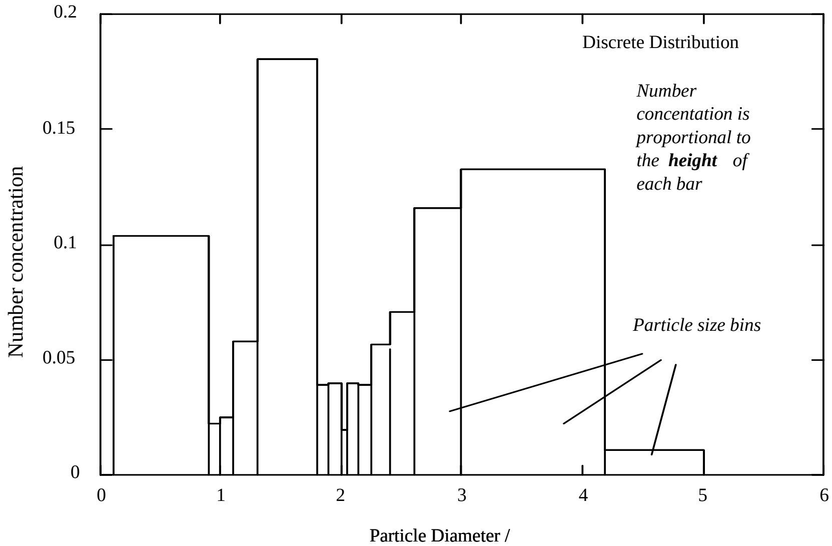
Concentation versus particle size

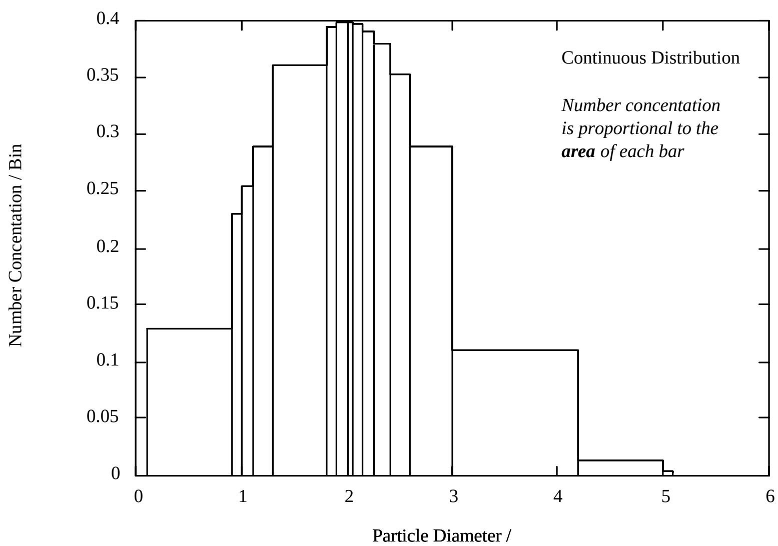
Aerosol particle sizing

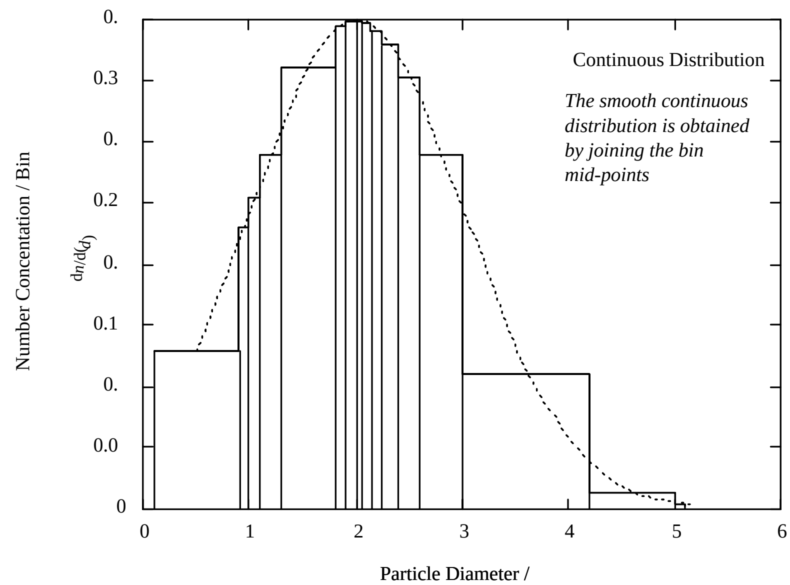


Particles are assigned to bins according to particle diameter

Conversion of a discrete particle size distribution to a continuous distribution







Continuous graph

- Y-axis
  - Differential particle concentration (or number)
    - Particle number normalized by range of particle diameter of the interval (or bin)
    - d*n*/d*d*
- X-axis
  - Particle diameter range of the interval
    - d*d*
- Integrated area under the curve=total # of particle

$$\int \frac{dn}{dd} dd = n$$

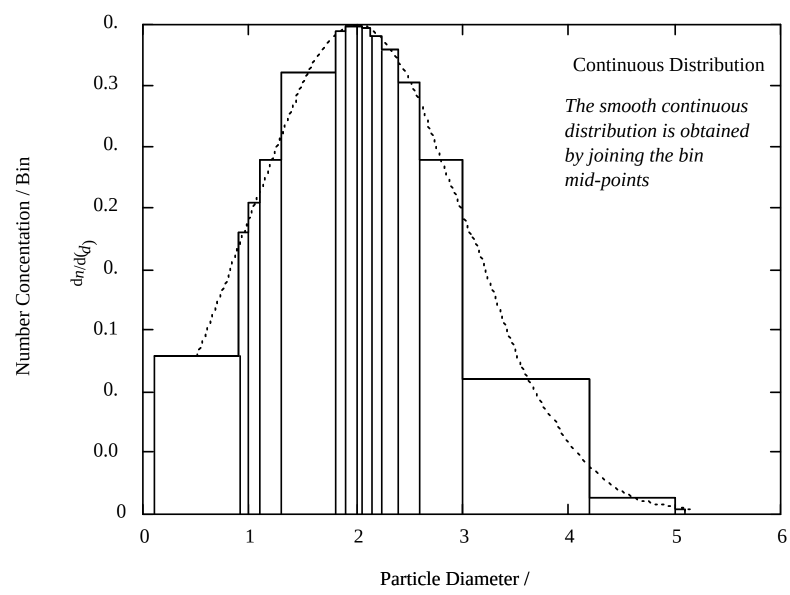
Properties of particle size distribution

- Asymmetrical (or skewed) distribution
  - Long tail to the right
    - Large number (fraction) of small particles
    - Small number (fraction) of large particles
  - Large range of particle size
    - Several orders of magnitude in particle diameter
  - No negative particle size
- Mode < Median < Mean
- Geometric mean (d_g or GM)

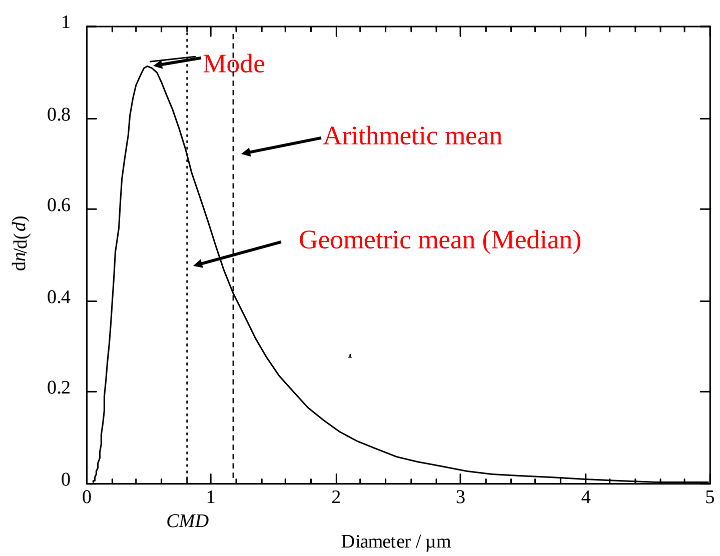
$$\text{Log } d_g = (\Sigma n_i \log d_i)/N \rightarrow d_g = \exp\{(\Sigma n_i \log d_i)/N\}$$

Arithmetic mean and GM

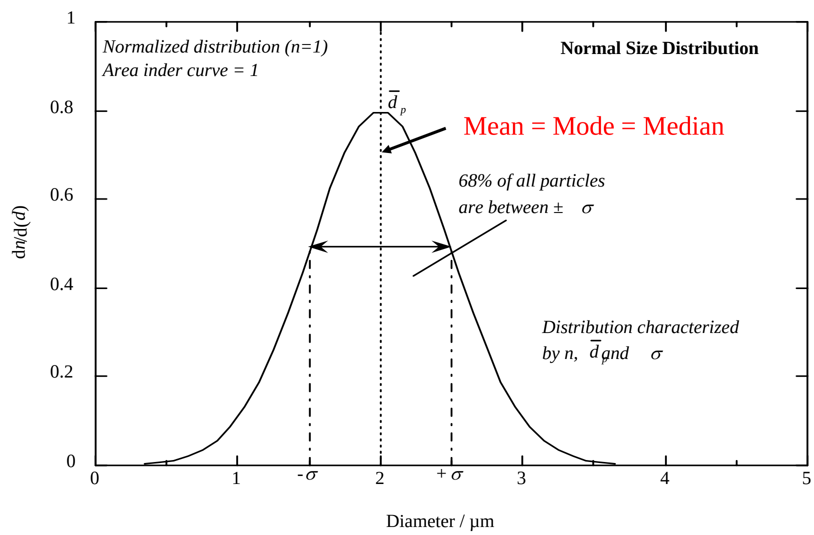
- Arithmetic mean
  - $d_a = (\Sigma n_i d_i)/N$ = (summation of all areas of the bars) / (total number of particles)
- Geometric mean
  - $\text{Log } d_g = (\Sigma n_i \log d_i)/N \rightarrow d_g = \exp\{(\Sigma n_i \log d_i)/N\}$



Asymmetric (skewed) distribution



Normal distribution



Normal distribution

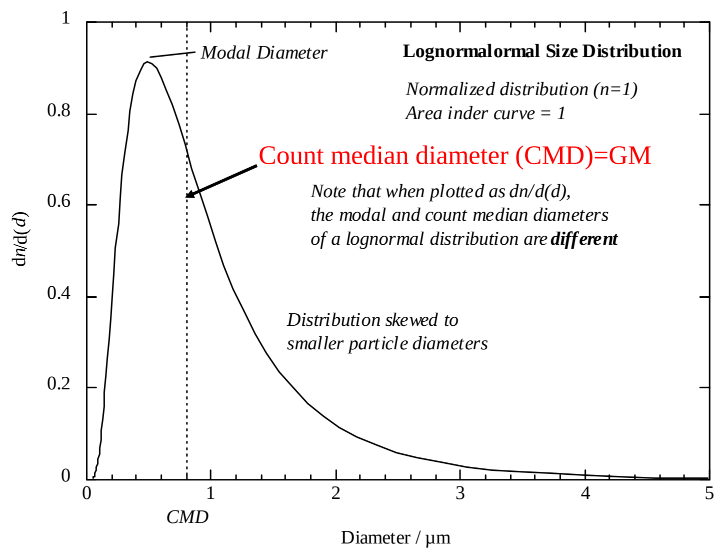
$$df = \frac{n}{\sigma\sqrt{2\pi}} e^{-\frac{(d_p - \bar{d}_p)^2}{2\sigma^2}} dd_p$$

- $\bar{d}p$: arithmetic mean particle diameter
- σ: standard deviation
- *ddp*: particle diameter interval
- *df*: frequency of occurrence of particles of diameter *dp*

Log transformation of continuous graph

- Y-axis
  - Differential particle concentration (or number)
    - Particle number normalized by range of log-transformed particle diameter of the interval (or bin)
    - d*n*/dlog(*d*)
- X-axis
  - Range of log-transformed particle diameter of the interval
    - dlog(*d*)
- Integrated area under the curve=total # of particle
  - $\int\{dn/d\log(d)\}d\log(d)$

Lognormal distribution with arithmetic scale



Mathematical function of lognormal distribution

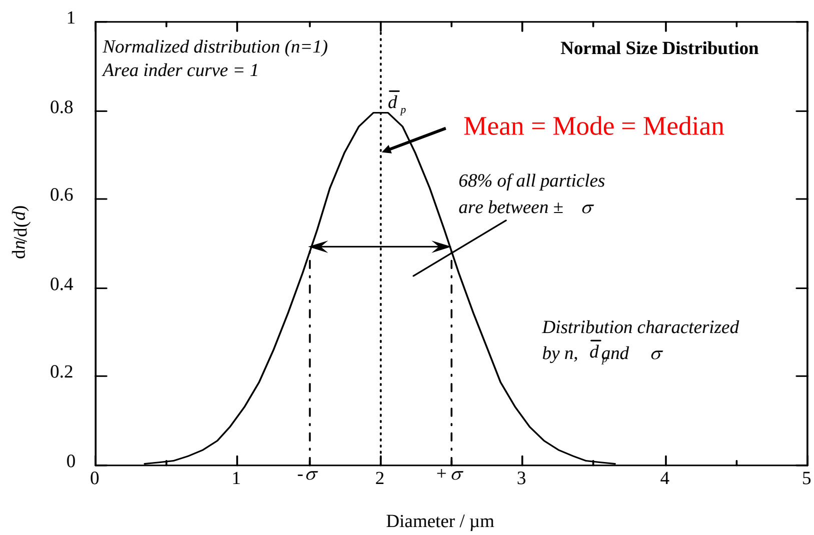
$$df = \frac{n}{\sqrt{2\pi}\, Log(\sigma_g)} e^{-\frac{(Log(d_p) - Log(CMD))^2}{2Log(\sigma_g)^2}} dLog(d_p)$$

$$df = \frac{n}{\sigma\sqrt{2\pi}} e^{-\frac{(d_p - \bar{d}_p)^2}{2\sigma^2}} dd_p$$

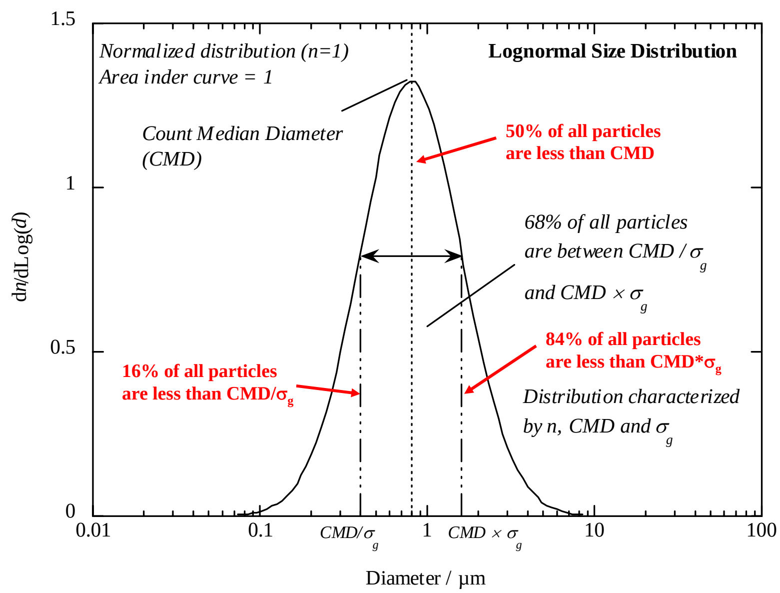
Two ways of log-transformed graph

- Transform the original particle size data using logarithm, and then plot them on normal arithmetic scale of the graph
  - To calculate all statistics mathematically
  - Exponentiate log-transformed statistics
- Transform x-axis scale of the graph, and then plot the original particle size data on it
  - Do not transform the data
  - Only change the scale of the graph

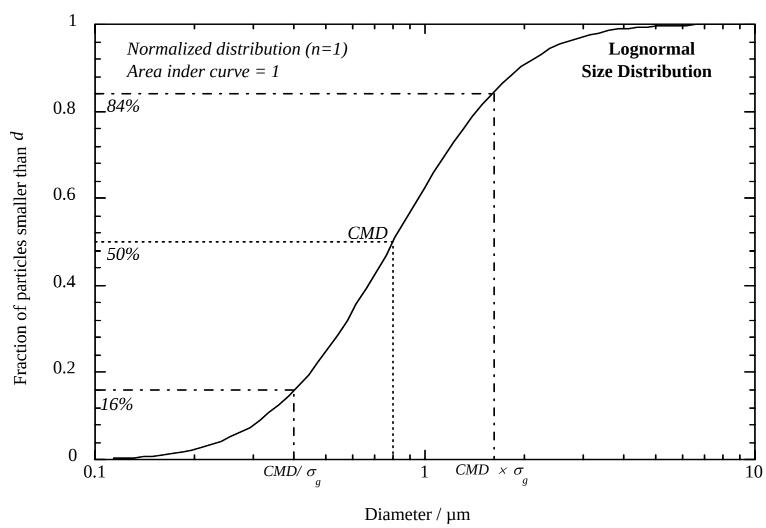
Log normal distribution (first approach)



Lognormal distribution with log scale



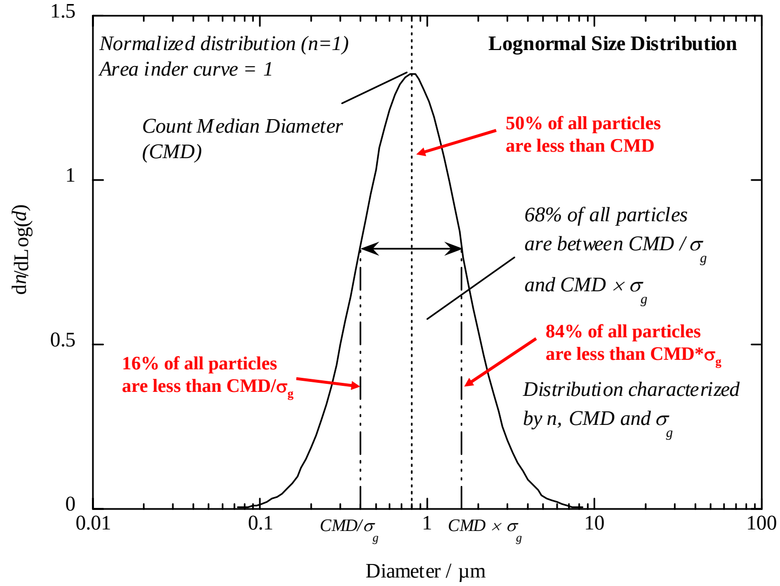
Cumulative size distribution



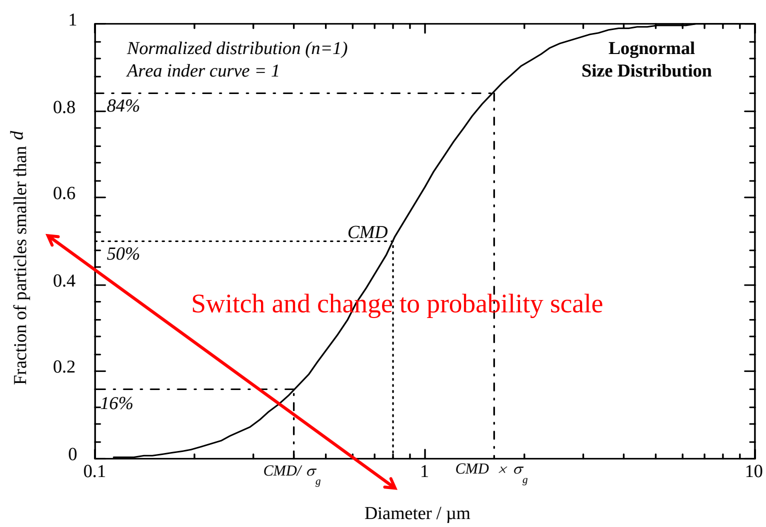
Probability scale of lognormal distribution

- Percent of particles less than a given particle diameter
  - <CMD/(2σ_g)- 5%
  - <CMD/σ_g- 16 %
  - <CMD (median)- 50%
  - <CMD*σ_g- 84%
  - <CMD*(2σ_g)- 95%
- Any pattern??
  - Symmetry of probability

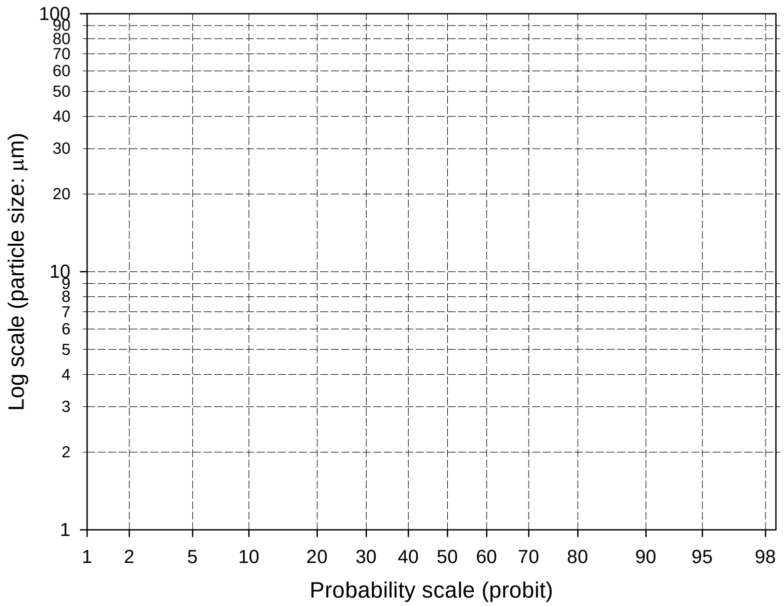
Lognormal distribution with log scale



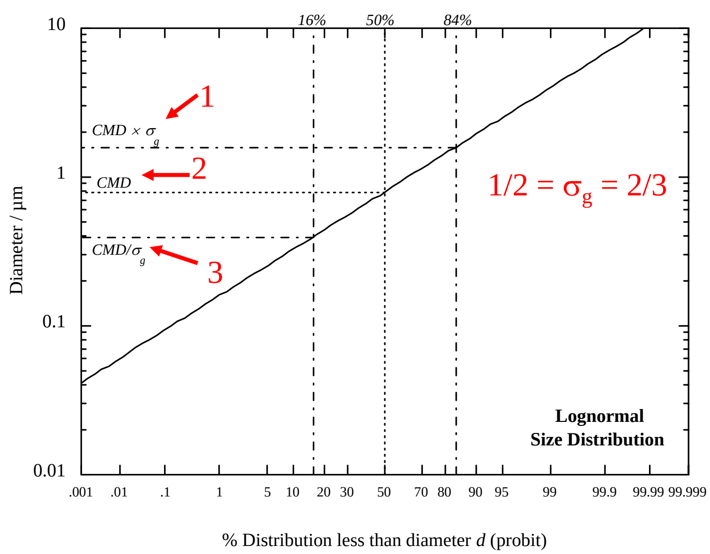
Cumulative plot to log-probability plot



Log-probability paper



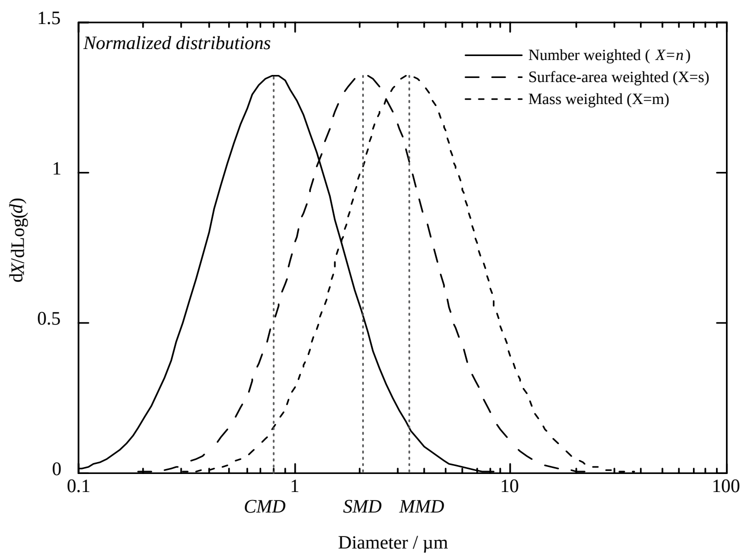
Log-probability plot of lognormal distribution



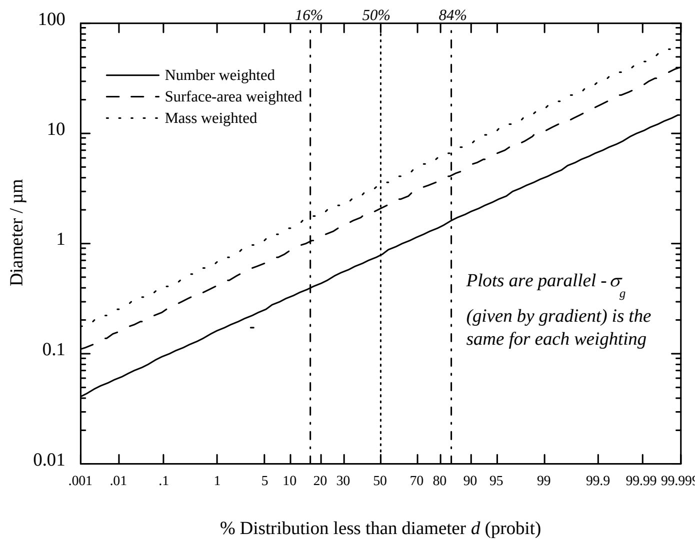
Count median diameter and σ_g

- CMD (count median diameter)
  - 50% of all particles are less than CMD
- Geometric standard deviation
  - CMD*σ_g/CMD or
  - CMD/(CMD/σ_g)

CMD, SMD, and MMD



Log-probability plots for count, mass, and surface area lognormal distribution



Cascade impactor data reduction

Stage #	Size Range (µm)	d_{50} (µm)	Initial Mass (mg)	Final Mass (mg)	**Net Mass (mg)**	**Mass Fraction (%)**	**Cumm. Mass Fraction (%)**
1	>9.0	9.0	850.5	850.6	**0.1**	**0.6**	**100.0**
2	4.0-9.0	4.0	842.3	844.1	**1.8**	**11.0**	**99.4**
3	2.2-4.0	2.2	855.8	861.0	**5.2**	**31.7**	**88.4**
4	1.2-2.2	1.2	847.4	853.6	**6.2**	**37.8**	**56.7**
5	0.7-1.2	0.70	852.6	855.1	**2.5**	**15.2**	**18.9**
Back filter	0-0.7	0	78.7	79.3	**0.6**	**3.7**	**3.7**
Total					**16.4**	**100.0**	

An example using count data

Data

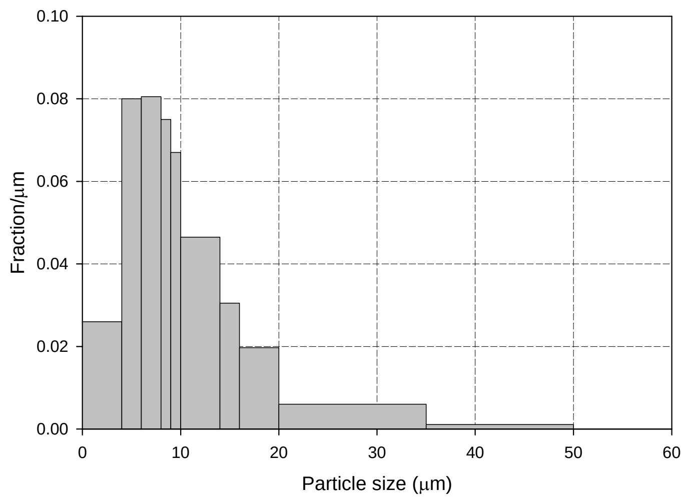
TABLE 4.1 Example of Grouped Data

Size Range[a] (μm)	Count	Fraction/μm	Percent	Cumulative Percent
0–4	104	0.026	10.4	10.4
4–6	160	0.080	16.0	26.4
6–8	161	0.0805	16.1	42.5
8–9	75	0.075	7.5	50.0
9–10	67	0.067	6.7	56.7
10–14	186	0.0465	18.6	75.3
14–16	61	0.0305	6.1	81.4
16–20	79	0.0197	7.9	89.3
20–35	90	0.0060	9.0	98.3
35–50	17	0.0011	1.7	100.0
>50	0	0.0	0.0	100.0
Total	1000		100.0	

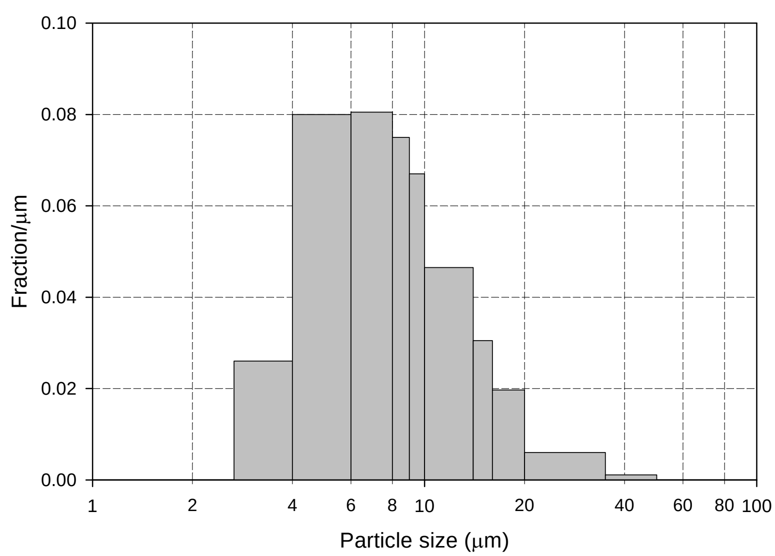
[a]Intervals are equal to or greater than the lower limit and less than the upper limit.

William Hinds, Aerosol Technology, 2nd (1999)

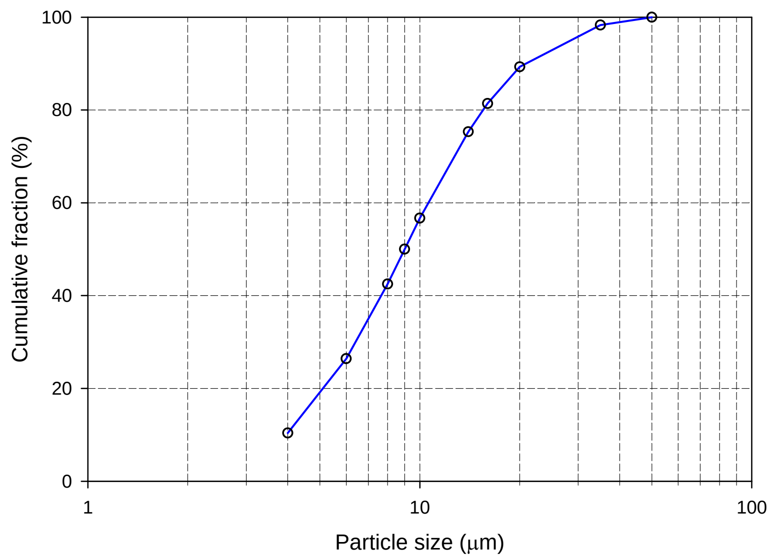
Plotting upper size range vs. fraction/μm



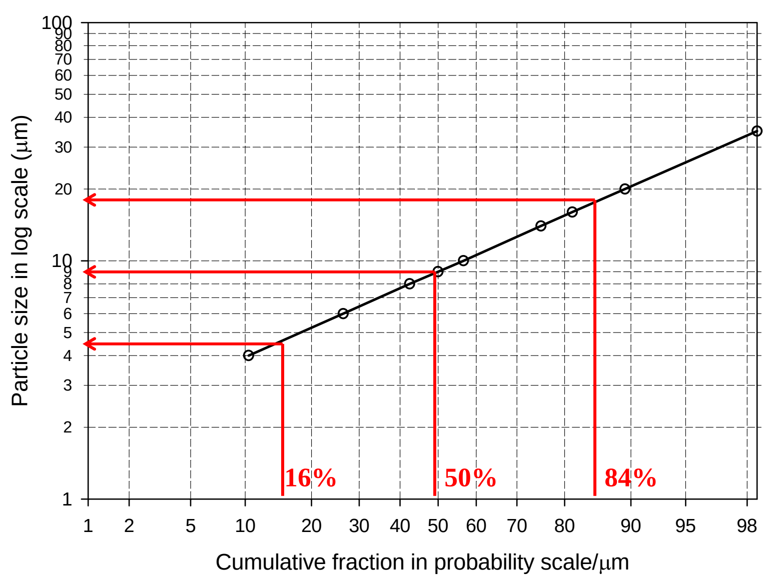
Log scale of particle diameter (lognormal distribution)



Cumulative lognormal plot (cum vs. size)



Log-probability plot of count data



Take-home practice

Stage cut-point (diameter d) (µm)	Mass concentration (mg/m^3)	Cumulative mass concentration greater than diameter d (mg/m^3)	% Cumulative mass concentration greater than diameter	% Cumulative mass concentration less than diameter
10	0.4209			
5.62	1.3330			
3.16	3.3770			
1.78	6.4771			
1	9.5248			
0.5	12.7579			
0.285	8.1861			
0.158	5.2389			
0.05	2.9853			

Sum (mg/m^3)	

1. Fill out the blank columns.
2. Find out MMD (Mass Median Diameter) and Geometric standard deviation (GSD).