Extra Problems (ungraded)
11/22/2005

1 Given the data below, determine whether the observed distribution is significantly different from the expected distribution at α = 5%

Range	0-1	1-2	2-3	3-4	4-5
Observed	13	20	27	14	26
Expected	20	20	20	20	20

Does have at least 5 values for each cell.

Obs-Exp	-7	0	7	-6	6
$(\text{Obs-Exp})^2$	49	0	49	36	36
$\chi^2 = \Sigma$	2.45	0	2.45	1.8	1.8
=	8.50				

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i}$$

df = $\upsilon = k - 1 =$ 4

$\chi^2_{0.05,4} =$ 9.488

$\chi^2 < \chi^2_{0.05,4}$ Not significant at 5%

Prob = 0.075 Not significant at 5%

2 Given the data below, determine whether the observed distribution is significantly different from the expected distribution at α = 5%

Range	0	1	2
Observed	32	15	13
Expected	28.32	21.24	10.44

Does have at least 5 values for each cell.

Obs-Exp	3.68	-6.24	2.56
$(\text{Obs-Exp})^2$	13.54	38.94	6.554
$\chi^2 = \Sigma$	0.478	1.833	0.628
=	2.939		

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i}$$

df = $\upsilon = k - 1 =$ 2

$\chi^2_{0.05,4} =$ 5.99

$\chi^2 < \chi^2_{0.05,2}$ Not significant at 5%

P(χ2> 2.94) = 0.23

Not significant at 5%

4 Given the data below, determine whether the observed distribution is significantly different from the expected distribution at $\alpha =$ 5%

Range	0	1	2	3	4
Observed	15	15	12	3	1
Expected	24	12	6	3	1

Doesn't have at least 5 values for each cell.

Range	0	1	2	3	4
Observed	15	15	16		
Expected	24	12	10		

Does have at least 5 values for each cell.

Obs-Exp	-9	3	6	0	0
$(\text{Obs-Exp})^2$	81	9	36	0	0
$\chi^2 = \Sigma$	3.375	0.75	3.6	0	0
=	7.725				

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i}$$

df = $\upsilon = k - 1 =$ 2

$\chi^2_{0.05,4} =$ 5.99

$\chi^2 > \chi^2_{0.05,2}$ Significant

P(χ2> 7.73) = 0.021

Significant

5 When should pairing be done?

i) When the pairs have something in common that other pairs don't have.

ii) When there is less variability between values within pairs than between different pairs.

6 When should you compute a pooled variance?

Variance(s) are assumed to be *equal/unequal* (circle one) equal

True variance(s) are *known/unknown* (circle one) unknown

7 When do you use the two equations below?

Variance(s) are assumed to be *equal/unequal* (circle one) unequal

True variance(s) are *known/unknown* (circle one) unknown

$$T = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)}}$$

$$\upsilon = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2}{n_2}\right)^2}{n_2 - 1}}$$

8 A type I error is when you ***accept/reject*** the null hypothesis when it is ***true/false*** . reject/true

9 A type II error is when you ***accept/reject*** the null hypothesis when it is ***true/false*** . accept/false

10 The symbol for the probability of a type I error is: ____ α

11 The symbol for the probability of a type II error is: ____ β

12 What is the effect of increasing sample size on (circle one):

α errors?	*increas*	decrease	decreases
β errors?	*increas*	decrease	decreases

13 The ________ of a test is the probability of rejecting Ho given that a specific alternative is true. It is equal to 1-β. power

14 I wish to know with 95% confidence whether studying more will improve a student's test scores from the second statistics test to the third. I randomly select some students from the class and forced them to study more the second time. The results were:

Student:	1	2	3	4	5	X	s
2nd Test	77	89	89	54	82	78.2	14.4
3rd Test	82	78	94	77	87	83.6	7.0
Diff	5	-11	5	23	5	5.4	12.0

H_o: $\mu_d = 0$

H_1: $\mu_d > 0$

$$T = \frac{\bar{d} - \mu_{\bar{d}}}{s/\sqrt{n}} = \frac{5.4}{12.0 \quad 5^{0.5}} = 1.003447$$

$\nu = 4$

$t_{5\%,4} = 2.776$ not significant

$p <$ 0.186 not significant

15 We believe that product A and B have the same weight, but we want to sample to be sure. We wish to be 95% sure before we reject the belief that they are the same, and we want to be 90% sure we would reject equality if the true difference was truly 0.75 kg. We believe the standard deviations are 1.2 kg and 1.6 kg. How many samples should we take?

Questions: $\mu_d = \mu_A - \mu_B$

1 Null hypothes Ho: $\mu_d = 0$

2 Alternate hypothesis, H1: μ_d <> 0

3 Specific alternate null hypothesis: $\mu_{d\_1} = 0.75$

4 One sided or two sided? two

5 $\alpha =$ 5%
$\beta =$ 10%
power = 90%

6 Minimum sample size =

$$n = \frac{\left(z_{\alpha/2} + z_{\beta}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2}$$

$$= \frac{(1.96 + 1.282)^2 * 1.2^2 + 1.6^2}{0.75^2}$$

= 74.72
= 75

16 John alleges that the error in weighing product A is less than 4.3 kg. We wish to be 98% sure before we reject his claim. We also want to be 95% sure we would reject equality if the μ was 5.05 kg.
We believe the standard deviation is 2 kg.
How many samples should we take?

Questions:

1 Null hypothes Ho: $\mu = 4.3$

2 Alternate hypothesis, H1: $\mu < 4.3$

3 Specific alternate null hypothesis: $\mu_1 = 5.05$

4 One sided or two sided? one

5 $\alpha =$ 2%
$\beta =$ 5%
power = 95%

6 Minimum sample size =

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(2.054 + 1.644854)^2 * 2^2}{0.75^2}$$

= 97.28
= 98

17 John alleges that the average error in weighing product A = 0 kg.
We wish to be 99% sure before we reject that claim and we want to be
80% sure we would reject his claim if the true error v 1.1 kg.
We believe the standard deviation is 1.5 kg.
How many samples should we take?

Questions:

1 Null hypothes Ho: $\mu = 0$

2 Alternate hypothesis, H1: $\mu <> 0$

3 Specific alternate null hypothesis: $\mu_1 = 1.1$

4 One sided or two sided? two

5 $\alpha =$ 1%
$\beta =$ 20%
power = 80%

6 Minimum sample size =

$$n = \frac{\left(z_{\alpha/2} + z_{\beta}\right)^2 \sigma^2}{\delta^2}$$

$$\frac{(2.576 + 0.841621)^2 * (1.5)^2}{1.1^2}$$

= 21.72
= 22

18 We believe that product A weighs than B, but we want to sample to be sure.
We wish to be 98% sure before we reject the belief that they are
the same, and we want to be 80% sure we would reject equality if the
true difference is 1 kg. We believe the standard deviations are
2 kg and 1.1 kg. How many samples should we take?

Questions: $\mu_d = \mu_A - \mu_B$

1 Null hypothes Ho: $\mu_d = 0$

2 Alternate hypothesis, H1: $\mu_d > 0$

3 Specific alternate null hypothesis: $\mu_{d\_1} = 1$

4 One sided or two sided? one

5 $\alpha =$ 2%
$\beta =$ 20%
power = 80%

6 Minimum sample size =

$$n = \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{\delta^2}$$

$$= \frac{(2.054 + 0.842)^2 * (2^2 + 1.1^2)}{1^2}$$

= 43.68
= 44

20 Do a greater percentage of dogs attack their owners than cats? A survey [made up] found the following results: [use α = 5%]

omit

		Attacked	Total	phat
1	Cats	5	20	0.25
2	Dogs	10	23	0.434783

H_o: $p_{dogs} - p_{cats} = 0$

H_1: $p_{dogs} - p_{cats} > 0$

$$Z = \frac{p_{dogs} - p_{cats} - 0}{\left[\frac{p1q1}{n1} + \frac{p2q2}{n2}\right]^{0.5}} = \frac{0.184783}{\left[0.02006\right]^{0.5}} = 1.304667$$

P(p-p > 0) = 0.904

P(Z > above) = 0.096
Cannot reject null hypothesis. Alternate not proven.

21 Claimed that new spot remover removes 70% of spots. To check, will be used on 12 spots chosen at random. If x < 11, will accept null Ho: p=0.7
Otherwise, conclude p > 0.7.

a) Evaluate alpha, assuming p = 0.7 omit

Ho: p = 0.7 H1: p > 0.7

p= 0.7 n= 12 x = 11

α = P[X $\geq$ 11| p = 0.7]
= 1- P[X < 11| p = 0.7]
= 1 - 0.915
= **0.0850**

22 Random sample of 400 voters asked if favor a tax. If within range below, will conclude
p = 0.6 acceptance range: 220 < x < 260 omit

a) Find prob of type I error if actual p = 60%

p= 0.6 n= 400 μ = 240 σ = 9.797959

a) Find alpha
Using normal approximation Note that since boundaries not included, shrink them by 0.5

HIGH	$\underline{x}$ = 259.5	Z >	1.99	P(Z) =	0.977
LOW	$\underline{x}$ = 220.5	Z <	-1.99	P(Z) =	0.023

binomial

α = 0.023 + 0.023 0.030 0.022654
= **0.047**

23 Soft drink quantity supposed to have norm distr with mean = 200 ml and a std dev = 15 ml. Check with sample of 9 drinks. If $\underline{x}$ in the range below, then okay. Otherwise, conclude x <> 200 ml.
191 < x < 209 n = 9

a) Find alpha when μ = 200 ml

n= 9 μ = 200 σ = 15 σ_x = 5

low	$\underline{x}$ = 191	Z < -1.8	P(Z) = 0.036
high	$\underline{x}$ = 209	Z > 1.8	P(Z) = 0.036

α = 0.036 + 0.036
= **0.072**

b) What is prob of type II error if actual μ = 215
omit

n= 9 μ = 215 σ_x = 5

low	$\underline{x}$ = 191	Z >	-4.8	P(Z) =	0.000
high	$\underline{x}$ = 209	Z <	-1.2	P(Z) =	0.115

β = 0.115 - 0.000
= **0.115**

23 New cement has compressive strength of 5000 kg/cm2 and a standard dev = 120

Ho: μ = 5000 H$_1$: μ < 5000 n = 50

Critical region defined to be: $\underline{x}$ < 4970

a) Find alpha when Ho is true.

x $\leq$ 4970 n = 50

n= 50 μ = 5000 σ = 120 σ_x = 16.97056

μ_0 = 4970 Z < -1.77 P(Z) = 0.039

α = **0.039**

b) What is prob of type II error if actual μ = 4970 and 4960

n= 50 μ = 4970 σ_x = 16.97056

low	$\underline{x}$ = 4960	Z < -0.59	P(Z) = 0.278	β =	0.278
high	$\underline{x}$ = 4970	Z > 0	P(Z) = 0.500	β =	0.500

24 Light bulbs have normally distributed lifetime with mean and std deviation shown below.

$\mu = 800$ $\sigma = 40$ n = 30 $\underline{x}$ = 788

Test hypo below: Show P-value

H$_o$: $\mu = 800$

H$_1$: $\mu <> 800$

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{788 - 800}{40 / \sqrt{30}} = \frac{-12}{7.302967} = -1.643168$$

P(Z<-1.643)+ P(Z>1.643) = 0.05 + 0.05 = 0.10

25 Mice live to be about 40 months old with special diet.
Given statistics below, do you believe Ho below?

$\mu = 40$ n = 64 $\underline{x}$ = 38 $\sigma = 5.8$

Test hypo below: Show P-value

H_o: $\mu = 40$

H_1: $\mu < 40$

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{38 - 40}{40 / \sqrt{64}} = \frac{-2}{0.725} = -2.758621$$

P(Z<-2.76) = 0.003

26 Claimed autos driven mean given below. A random test had the statistics below.
Would you agree?

$\mu = ???$ s = 3900 n = 100 $\underline{x}$ = 23500

Test hypo below: Show P-value

H_o: $\mu = 22800$

H_1: $\mu > 22800$

$$Z = \frac{\overline{X} - \mu}{s / \sqrt{n}} = \frac{700}{390} = 1.794872$$

P(Z) = 0.0363
Reject Ho: mean > 20,000 km

27 Given the data below, test the Ho below.

Volumes:						$\underline{x}$	s
	10.1	9.7	10.1	10.2	10.1	10.0	0.200
	9.8	9.8	10.2	10.2	9.8		

n = 10 $\quad \underline{x}$ = 10 $\quad$ s = 0.2

Test hypo below: $\alpha = 0.01$

H_o: $\mu = 9.9$

H_1: $\mu <> 9.9$

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{0.10}{0.063} = 1.581139$$

P(T>t)+ P(T<-t) = 0.148 Do not reject Ho

$t_{.01,4}$ = 3.25 Do not reject Ho

28 Running increases basal metabolic rate for older women. Assume equal variances.

control	μ =	n = 10	$\underline{x}$ = 100	s = 32
exercise	μ =	n = 15	$\underline{x}$ = 134	s = 38

Test hypo below: $\alpha = 0.01$

H_o: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 > 0$

$$s_p^2 = \frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2} = \frac{1024 \quad 9 \quad + \quad 1444 \quad 14}{10 \quad + \quad 15 \quad - \quad 2} = 1280$$

s_p = 35.77

$$T = \frac{(\overline{X}_1 - \overline{X}_2)}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{134 \quad - \quad 100 \quad - \quad 0}{35.77 \quad (\quad 0.1 \quad + \quad 0.066667 \quad)^{0.5}} = 2.328$$

P(Z > 2.328) = 0.010 Reject Ho, accept H1

$Z_{0.01}$ = -2.326 Reject Ho, accept H1

29 Mfg claims that strength of thread A stronger than thread B by at least 12%.
Given statistics below, evaluate his claim at alpha. Do not assume equal variances.

A	μ =	n = 50	$\underline{x}$ = 87	s = 6.28
B	μ =	n = 50	$\underline{x}$ = 77	s = 5.61

Test hypo below: $\alpha = 0.05$

H_o: d = 12

H_1: d < 12

$$T = \frac{(\overline{X}_1 - \overline{X}_2) - d_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{10 - 12}{(0.789 + 0.629442)^{0.5}} = -1.67942$$

Large N, use Z

P(Z < -1.68) = 0.047 << 0.05

Reject Ho

30 Study to determine if increasing the substrate conc has appreciable effect
on the velocity of a chemical reaction. Results given below. Assume equal variances.

A=1.5	μ =	n = 15	$\underline{x}$ = 7.5	s = 1.5
B=2.0	μ =	n = 12	$\underline{x}$ = 8.8	s = 1.2

Test hypo below: $\alpha = 0.01$

H_o: d = 0.5

H_1: d > 0.5

$$s_p^2 = \frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2} = \frac{2.25 \quad 14 \quad + \quad 1.44 \quad 11}{15 \quad + \quad 12 \quad - \quad 2} = 1.8936$$

s_p = 1.376

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - d}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{8.8 - 7.5 - 0.5}{1.376 \; (0.066667 + 0.083333)^{0.5}} = 1.50107$$

P(t > 1.501) = 0.067 > 0.01

Do not reject Ho

$t_{0.01,25}$ = 2.479

31 Serum to treat leukemia in mice. Survival times given below. Assume normal with equal variances.

						x	s	sp
Treatment	2.1	5.3	1.4	4.6	0.9	2.86	1.970533	1.619756
No treat:	1.9	0.5	2.8	3.1		2.075	1.167262	

A=1.5 $\mu =$ n = 5 $\underline{x}$ = 2.86 s = 1.9705329

B=2.0 $\mu =$ n = 4 $\underline{x}$ = 2.075 s = 1.1672618

Test hypo below: $\alpha = 0.05$

H_o: d = 0

H_1: d > 0

$$s_p^2 = \frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2} = \frac{3.883 \quad 4 \quad + \quad 1.3625 \quad 3}{5 \quad + \quad 4 \quad - \quad 2} = 2.80279$$

s_p = 1.674

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - d}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{2.86 \quad - \quad 2.075 \quad - \quad 0}{1.674 \quad (\quad 0.2 \quad + \quad 0.25 \quad)^{0.5}} = 0.69899$$

P(t > 0.699) = 0.242 > 0.05

Do not reject Ho

$t_{0.05,7}$ = 1.895 0.05

32 Data represent running times of movies. Is one greater than the other? Assume normal but unequal variances.

								x	s	s^2
Co.1	102	86	98	109	92			97.40	8.9	78.8
Co.2	81	165	97	134	92	87	114	110.00	30.2	913.333

A=1.5 $\mu =$ n = 5 $\underline{x}$ = 97.4 s = 8.8769364

B=2.0 $\mu =$ n = 7 $\underline{x}$ = 110 s = 30.221405

Test hypo below: $\alpha = 0.1$ $\underline{d}$ = 12.6

H_o: d = 10

H_1: d > 10

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - d_0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{12.6 \quad - \quad 10}{(\ 15.76 \quad + \quad 130.4762\ )^{0.5}} = 0.215$$

P(t' < 0.215) = 0.585 >> 0.1 should use t and compute deg freedom the hard way.

Cannot reject Ho

33 Taxi manager trying to decide which tires to use for best gas mileage with 95% confidenc
Same cars and drivers tested each tire, with following results:

Car	1	2	3	4	5	6	7	8	9	10	11
adial	4.2	4.7	6.6	7	6.7	4.5	5.7	6	7.4	4.9	6.1
elted	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6
Diff	0.1	-0.2	0.4	0.1	-0.1	0.1	0	0.2	0.5	0.2	0.1

A=1.5	μ = ???	n = 12	x = 5.8	s = 1.0890363
B=2.0	μ = ???	n = 12	x = 5.672727	s = 1.0159635

Test hypo below: α = 5% d = 0.127273 s_d = 0.200454

H_o: d = 0

H_1: d < 0

$$T = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{0.127}{0.058} = 2.199435$$

P(t) = 0.050 Reject Ho. Radial tires better.

$t_{0.05,10}$ = 1.81 < 2.199 Reject Ho. Radial tires better.

34 I wish to know whether studying more will improve a student's test scores
from the second statistics test to the third. I randomly select 8 students from the class
and forced them to study more the second time. The results were:

Student:	1	2	3	4	5	X	s
2nd Test	77	89	89	54	82	78.2	14.4
3rd Test	82	78	94	77	87	83.6	7.0
Diff	5	-11	5	23	5	5.4	12.0

H_o: d = 0

H_1: d > 0

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} \quad \frac{5.4}{4.913} = 1.099221$$

ν = 4

α = 0.051

35 We believe that product A and B have the same weight, but we want to sample to be sure. We wish to be 95% sure before we reject the belief that they are the same, and we want to be 90% sure we would reject equality if the true difference was truly 0.75 kg. We believe the standard deviations are 1.2 kg and 1.6 kg. How many samples should we take?

Questions: $\mu_d = \mu_A - \mu_B$

1 Null hypothes Ho: $\mu_d = 0$

2 Alternate hypothesis, H1: $\mu_d <> 0$

3 Specific alternate null hypotl $\mu_{d\_1} = 0.75$

4 One sided or two sided? two

5 $\alpha =$ 5%

$\beta =$ 10%

power = 90%

6 Minimum sample size =

$$n = \frac{\left(z_{\alpha/2} + z_\beta\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2}$$

$$= \frac{(1.96 + 1.282)^2 \quad 1.2^2 + 1.6^2}{0.75^2}$$

= 74.72

= 75

36 John alleges that the error in weighing product A is less thar 4.3 kg. We wish to be 98% sure before we reject his claim. We also want to be 95% sure we would reject equality if the ı was 5.05 kg. We believe the standard deviation is 2 kg. How many samples should we take?

Questions:

1. Null hypothes Ho: $\mu = 4.3$
2. Alternate hypothesis, H1: $\mu < 4.3$
3. Specific alternate null hypotl $\mu_1 = 5.05$
4. One sided or two sided? one
5. $\alpha =$ 2%
   $\beta =$ 5%
   power = 95%
6. Minimum sample size =

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(2.054 + 1.644854)^2 \; 2^2}{0.75^2}$$

$= 97.28$

$= 98$

37 John alleges that the average error in weighing product A = 0 kg.
We wish to be 99% sure before we reject that claim and we want to be
80% sure we would reject his claim if the true error v 1.1 kg.
We believe the standard deviation is 1.5 kg.
How many samples should we take?

Questions:

1 Null hypothes Ho: $\mu = 0$

2 Alternate hypothesis, H1: $\mu <> 0$

3 Specific alternate null hypotl $\mu_1 = 1.1$

4 One sided or two sided? two

5 $\alpha =$ 1%
$\beta =$ 20%
power = 80%

6 Minimum sample size =

$$n = \frac{\left(z_{\alpha/2} + z_{\beta}\right)^2 \sigma^2}{\delta^2} \quad \frac{(2.576 + 0.841621)^2 \; 1.5^2}{1.1^2}$$

= 21.72
= 22

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38 We believe that product A weighs than B, but we want to sample to be sure.
We wish to be 98% sure before we reject the belief that they are
the same, and we want to be 80% sure we would reject equality if the
true difference is 1 kg. We believe the standard deviations are
2 kg and 1.1 kg. How many samples should we take?

Questions: $\mu_d = \mu_A - \mu_B$

1 Null hypothes Ho: $\mu_d = 0$

2 Alternate hypothesis, H1: $\mu_d > 0$

3 Specific alternate null hypotl $\mu_{d\_1} = 1$

4 One sided or two sided? one

5 $\alpha =$ 2%
$\beta =$ 20%
power = 80%

6 Minimum sample size =

$$n = \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{\delta^2}$$

$$= \frac{(2.054 + 0.842)^2 \quad 2^2 + 1.1^2}{1^2}$$

= 43.68
= 44

39 Thought that 40% would choose lasagna. Sampling statistics shown below. Use alpha 5%
Show P-value

x = 9 n = 20 $p_{hat} =$ 0.45

H_o: p = 0.4 8
H_1: p > 0.4

$$P[X \geq 9 \mid p = 0.4] = 1 - \sum_{0}^{8} b(x;20,0.4) = \quad 0.4044$$

H_o: p = 0.4
H_1: p <> 0.4
for fun, show boundaries if used critical values
$P(X \leq 4 \mid p = 0.4) + P(X \geq 16 \mid p = 0.$ 0.050952 + 4.73E-05
= 0.050999

40 Coin tossed 20 times, resulting in 5 heads. Is the coin equally balanced? Give P-value.

x = 5   n = 20 p_{hat} = 0.25

H_o: p = 0.5   10

H_1: p < 0.5

$$P[X < 10 \mid p = 0.5] = \sum_{0}^{5} b(x;20,0.5) = \quad 0.02069$$

Reject Ho. Accept H1

41 Claimed that 20% of homes heated by oil. Should we doubt this if survey shows 136/1000 heated by oil. Show P-value.

x = 136   n = 1000 p_{hat} = 0.136

$\mu = np =$ 200

$\sigma^2 = npq =$ 160

$\sigma =$ 12.64911

H_o: p = 0.2

H_1: p < 0.2

$$Z = \frac{\overline{X} - \mu}{\sigma_p} = \frac{64}{12.65} = 5.059644$$

P(Z) = #####

Reject Ho: Accept H1

42 Radar successful or not. If successful in 250/300 trials. At alpha = 4%, consider claim that success $\leq$ 0.8.

x = 250   n = 300 p_{hat} = 0.833

$\mu = np =$ 240

$\sigma^2 = npq =$ 48

$\sigma =$ 6.928203

H_o: p = 0.8

H_1: p < 0.8

$$Z = \frac{\overline{X} - \mu}{\sigma_p} = \frac{10}{6.928} = 1.443376$$

P(X<240) = 0.074

Cannot reject Ho, but should do more trials.

43 Survey for those in favor of nuclear power plant. 63/100 urban favored, while 59/125 suburban favored. Is there a significant diff between urban and suburban opinion? Use p-value.

Urban	x = 63	n = 100	$p_{hat} =$	0.630
Suburban	x = 59	n = 125	$p_{hat} =$	0.472
		total =	$p_{hat} =$	0.542222

H_o: $p_u = p_s$

H_1: $p_u <> p_s$

$$Z = \frac{\left(\hat{P}_1 - \hat{P}_2\right) - 0}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.63 - 0.472)}{\sqrt{0.542(1 - 0.542)\left(\frac{1}{100} + 125\right)}} =$$

$$= \frac{0.158}{0.067} = 2.363768$$

P(Z < -2.36) + P(Z>2.36) = 0.0181

Reject Ho. Accept H1

44 City thinks breast cancer rate higher than nearby rural area.
Can we conclude at alpha = 0.05 level that urban is worse?

	x	n
Urban	20	200
Rural	10	150

Urban	x = 20	n = 200	$p_{hat} =$ 0.1
Rural	x = 10	n = 150	$p_{hat} =$ 0.0666667
		total =	$p_{hat} =$ 0.085714

H_o: $p_u = p_r$
H_1: $p_u > p_r$

$$Z = \frac{\left(\hat{P}_1 - \hat{P}_2\right) - 0}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.1 - 0.0667)}{\sqrt{0.086\,(1 - 0.086)\left(\frac{1}{200} + \frac{1}{150}\right)}} =$$

$$= \frac{0.033}{0.03} = 1.102396$$

P(Z >0.93) = 0.1351
Cannot reject Ho.

45 Lubricant volume normally distr with variance shown below. Test the hypo that the expected variance below for a random sample of n. Use P-value.

Data:					
	10.2	9.7	10.1	10.3	10.1
	9.8	9.9	10.4	10.3	9.8

n = 10 claimed $\sigma^2 = 0.03$ $s^2 = 0.060444$

H_o: $\sigma^2 = 0.03$ $\alpha = 5\%$
H_1: $\sigma^2 <> 0.03$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_o^2} = \frac{9 \quad 0.060444}{0.03}$$

$$= 18.13$$

2P(χ^2 : 18.13) = 0.034 Can't reject

$\chi^2_{2.5\%,9} = 19.02 < 18.13$ Can't reject

46 Aflotoxins in peanuts. Sample shows values below. Test Ho below.

$n = 64$ $\sigma^2 = 4.2$ $s^2 = 4.25$

H_o: $\sigma^2 = 4.2$ $\alpha = 5\%$
H_1: $\sigma^2 <> 4.2$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_o^2} = \frac{63 \quad 4.25}{4.2} = 63.75$$

$2P(\chi^2 > 63.75) = 0.9$ Cannot reject.

$\chi^2_{2.5\%,9} = 86.83 < 63.75$ Can't reject 0.025

47 Out of control if variance exceeds Ho below. Random sample data shown below.
shown below. Is this out of control at alpha = 0.05, assuming normally distributed?

$n = 25$ $\sigma^2 = 1.15$ $s^2 = 2.03$

H_o: $\sigma^2 = 1.15$ $\alpha = 5\%$
H_1: $\sigma^2 > 1.15$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_o^2} = \frac{24 \quad 2.03}{1.15}$$

$= 42.37$

$P(\chi^2 > 42.37) = 0.012$ Reject.

$\chi^2_{2.5\%,24} = 39.36 < 42.37$ Can't reject 0.025

48 Comparing time required by men and women for a task. Variance of women may be < men.
Random sample produced values below. Is variance higher for men (Use P-value)?

men	$n = 11$	$\sigma^2 = ???$	$s = 6.1$
women	$n = 14$	$\sigma^2 = ???$	$s = 5.3$

H_o: $\sigma^2 = \sigma^2$ $\alpha = 5\%$
H_1: $\sigma^2 > \sigma^2$

$$F = \frac{s^2}{s^2} = \frac{37.21}{28.09} = 1.324671$$

$P(F_{10,13} > 1.33) = 0.312$ Cannot reject.

$f_{5\%,10,14}$ = 2.671 > 1.325 Can't reject 0.05

49 For hydrocarbon emissions we wish to compare 80's and 90's cars.
Random sample produced values below. Is variance different for the sets of cars?
Use P-values.

1980	141	359	247	940	882	494	306	210	105	880
	220	223	188	940	241	190	300	435	241	380
1990	140	160	20	20	223	60	20	95	360	70
	220	400	217	58	235	380	200	175	85	65

1980 n = 20 σ^2 = ??? s = 280.3708
1990 n = 20 σ^2 = ??? s = 119.3946

H_o: $\sigma^2 = \sigma^2$ α = 2%
H_1: $\sigma^2 <> \sigma^2$

$$F = \frac{s^2}{s^2} = \frac{78608}{14255} = 5.514369$$

$P(F_{19,19} >$ 5.514) = 5E-04 Reject.

$f_{1\%,19,19}$ = 2.645 < 5.514 Reject 0.02

50 A die is tossed with the following results:
Is this a balanced die? Use alpha = 0.01

x	1	2	3	4	5	6	Total	
f	28	36	36	30	27	23	180	
expected	30	30	30	30	30	30	180	
$(Obs-Exp)^2/Exp$	0.133	1.2	1.2	0	0.3	1.633333	4.466667	<-- Sum of relative differences

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i} = 4.467$$

$\chi^2_{0.01,\ 5}$ > 15.09 > 4.467 Can't reject
Cannot reject Ho: die is balanced.

51 Three cards are drawn at random from an ordinary deck of cards with replacement. The number, Y, of spades is recorded. The results for 64 times are shown below. Determine whether a binomial distr of b(x; 3, 1/4) fits the results at alpha = 0.01

p= 0.25
n = 3

x	0	1	2	3		
f	21	31	12	0	64	
expected	27	27	9	1	64	from binomial b(x; 3, 1/4)

less than 5

x	0	1	2&3		
f	21	31	12	64	
expected	27	27	10		
$(Obs-Exp)^2/Exp$	1.33	0.59	0.40	2.326	Sum of relative differences

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i} = 2.326$$

$\chi^2_{0.01,\ 2}$ > 9.21

Cannot reject Ho: follows binomial

52 A coin is tossed until a head occurs and the number X of tosses recorded. After repeating 256 times, obtain the following results. Test the hypothesis at the 0.05 level that the observed distribution can be fitted by the hypergeometric distr. alpha = 0.05 256

x	1	2	3	4	5	6	7	8	Total
freq	136	60	34	12	9	1	3	1	256
expected freq	128	64	32	16	8	4	2	1	255
consolidated freq	136	60	34	12	9	5			256
g(x,p)*N=	128	64	32	16	8	7			255
(O-E)/E	0.5	0.25	0.125	1	0.125	0.571429			2.571429

$g(x,p) = p\, q^{x-1}$

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i} = 2.571$$

$\chi^2_{0.05,\ 5}$ > 11.07

Cannot reject Ho

53 A random sample of 90 adults are classified according to gender and the number of hours they watch TV during the week: alpha = 0.01

	Gender		
	Male	Female	Total
>25 hrs	15	29	44
<25 hrs	27	19	46
Total	42	48	90

Expected		
>25 hrs	20.53	23.47
<25 hrs	21.47	24.53

$(O\text{-}E)^2/E$

1.49	1.30
1.43	1.25

$$\text{expected contingency} = \frac{(\text{column total}) \times (\text{row total})}{\text{grand total}}$$

$$\chi^2 = \sum_{i=1}^{k} \frac{(Obs_i - Exp_i)^2}{Exp_i} = 5.47$$

$\chi^2_{0.01,\ 1}$ > 6.63

Cannot reject Ho