

Remember that the probability of anything is equal to the sum of the probabilities of its parts. This is most obvious when looking at complements:

$$P(A) = P(A \cap B) + P(A \cap B')$$

Since: $P(A \cap B) = P(A|B) \times P(B)$

And: $P(A \cap B') = P(A|B') \times P(B')$

Then: $P(A) = P(A|B) \times P(B) + P(A|B') \times P(B')$

The equation above can be solved for any of its terms if the others are known.

For example, the probability of being late to school could depend on missing your ride, but you can be late for other reasons, too:

$$P(\text{Late}) = P(\text{MissRide} \cap \text{Late}) + P(\text{MissRide}' \cap \text{Late})$$

$$P(\text{MissRide} \cap \text{Late}) = P(\text{MissRide}) \times P(\text{Late}|\text{MissRide})$$

$$P(\text{MissRide}' \cap \text{Late}) = P(\text{MissRide}') \times P(\text{Late}|\text{MissRide}')$$

$$P(\text{Late}) = P(\text{MissRide}) \times P(\text{Late}|\text{MissRide}) + P(\text{MissRide}') \times P(\text{Late}|\text{MissRide}')$$

Suppose you were asked the probability of being late given that you did not miss your ride: i.e., $P(\text{Late}|\text{MissRide}')$

$$P(\text{Late}) = 0.10$$

$$P(\text{MissRide}) = 0.15$$

$$P(\text{Late}|\text{MissRide}) = 0.40$$

Solution:

1) Find $P(\text{MissRide}') = 1 - P(\text{MissRide}) = 0.85$

2) $P(\text{Late}|\text{MissRide}') = \frac{P(\text{Late}) - P(\text{MissRide}) \times P(\text{Late}|\text{MissRide})}{P(\text{MissRide}')}$

$$= \frac{0.10 - 0.15 \times 0.40}{0.85} = \frac{0.04}{0.85} = 0.047$$

3) Does this value seem reasonable?

You would think the chances would be much lower if you caught your ride and it is. Also:

$$P(\text{Late}) = P(\text{MissRide}) \times P(\text{Late}|\text{MissRide}) + P(\text{MissRide}') \times P(\text{Late}|\text{MissRide}')$$

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