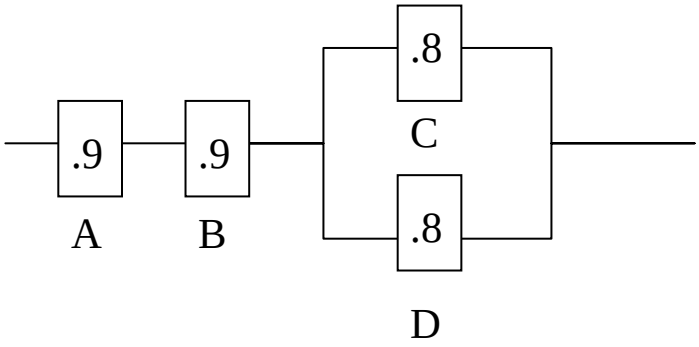
P(system') for the Circuit Problem in Chapter 2

In the PPT file for Chapter 2 I have a "circuit problem" (see Figure below).



It can be solved directly for the probability of success or for the probability of failure. As it turns out, solving for the probability of success is easier in this case.

The probability of failure is the union of all the ways it can fail:

$$P(\text{system}') = P(A' \cup B' \cup [C' \cap D'])$$

It is important to note that A, B, C, D, and E are independent, but they are NOT exclusive. If they were exclusive, it would be impossible for any two to fail at one time, and there is no reason to think that.

Hence: $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

Note that the intersection of A' and B' must be subtracted since because the intersection is counted in both P(A') and P(B').

Likewise, the left and right sides of the circuit are independent, not exclusive. Hence, the interactions must be subtracted from the union of the left and right. This is easiest to see if we substitute:

$$\begin{aligned} P(\text{Left}') &= P(A' \cup B') \\ &= P(A') + P(B') - P(A' \cap B') \\ &= 0.1 + 0.1 - 0.1 \times 0.1 = 0.19 \end{aligned}$$

$$\begin{aligned} P(\text{Right}') &= P(C' \cap D') \\ &= P(C') \times P(D') \\ &= 0.2 \times 0.2 = 0.04 \end{aligned}$$

$$\begin{aligned} P(\text{system}') &= P(\text{Left}' \cup \text{Right}') \\ &= P(\text{Left}') + P(\text{Right}') - P(\text{Left}' \cap \text{Right}') \\ &= 0.19 + 0.04 + 0.19 \times 0.04 \\ &= 0.224 \end{aligned}$$