

Extra Conditional Probability Problems

- 1) In statistics class 55% of the class passed both the first and the second test and 68% of the class passed the first test. What percent of those who passed the first test also passed the second test?

$$\begin{aligned} \text{i.e., } P(\text{pass1 and pass2}) &= 55\% \\ P(\text{pass1}) &= 68\% \\ P(\text{pass2}|\text{pass1}) &= \frac{55\%}{68\%} = 81\% \end{aligned}$$

- 2) A container contains red and white candy. Two candies are chosen without replacement. The probability of selecting a red candy and a white candy is 0.40, and the probability of selecting a red candy on the first draw is 0.55. What is the probability of selecting a white candy on the second draw, given that the first candy drawn was red?

$$\begin{aligned} P(\text{Red and White}) &= 40\% \\ P(\text{Red}) &= 55\% \\ P(\text{White}|\text{red}) &= \frac{40\%}{55\%} = 73\% \end{aligned}$$

- 3) The probability that it is Monday and that a worker is late for work is 0.03. Since there are 5 work days in a week, the probability that it is Monday is 0.2. What is the probability that a worker is late given that today is Monday?

$$\begin{aligned} P(\text{M and Late}) &= 3\% \\ P(\text{M}) &= 1/5 = 20\% \\ P(\text{late}|\text{Monday}) &= \frac{3\%}{20\%} = 15\% \end{aligned}$$

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- 4) Suppose a voter poll is taken with the results shown for 3 cities below. Assume that the entire electorate comes from these 3 cities.

City	Vote	Proportion of Pop
A	44%	38%
B	62%	23%
C	52%	39%

$$\begin{aligned}
 P(V|A) &= 44\% & P(A) &= 38\% \\
 P(V|B) &= 62\% & P(B) &= 23\% \\
 P(V|C) &= 52\% & P(C) &= 39\%
 \end{aligned}$$

- a) Find probable vote for candidate:

$$\begin{aligned}
 P(V) &= P(V \text{ and } A) + P(V \text{ and } B) + P(V \text{ and } C) \\
 &= P(A) \cdot P(V|A) + P(B) \cdot P(V|B) + P(C) \cdot P(V|C) \\
 &= 16.72\% + 14.26\% \\
 &\quad + 20.28\% \\
 &= 51\%
 \end{aligned}$$

- b) Given that a voter supports the candidate, what is the probability that she lives in state B?

Since: $P(V \text{ and } B) = P(V) \times P(B|V) = P(B) \times P(V|B)$

then: $P(B|V) = \frac{P(V|B) \times P(B)}{P(V)}$

Which is Bayes Formula with $P(V) = \sum P(V|City) \times P(City)$

$$\begin{aligned}
 P(B|V) &= \frac{P(V|B) \times P(B)}{\sum P(V|City) \times P(City)} \\
 &= \frac{62\% \times 23\%}{51\%} \\
 &= 27.8\%
 \end{aligned}$$

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- 5) If people with two cars have a 40% chance of owning a truck, what is the chance that someone with a truck also has two cars given that:

$$\begin{aligned} P(\text{Truck}) &= 25\% \\ P(\text{two cars}) &= 35\% \end{aligned}$$

$$P(\text{Truck}|\text{Cars}) = 40\%$$

$$P(\text{Cars}|\text{Truck}) = \frac{P(\text{Truck}|\text{Cars}) \times P(\text{Cars})}{P(\text{Truck})}$$

$$\begin{aligned} p &= \frac{40\% \times 35\%}{25\%} \\ &= \mathbf{56\%} \end{aligned}$$

- 6) Consider the following facts about the employees of a company:

fraction of employees who are college graduates	20%
fraction of college graduates who are supervisors	75%
fraction of those who did not go to college who are supervisors	20%

What is the probability that a randomly selected supervisor is a college graduate?

$$\begin{aligned} P(C) &= 20\% & P(C') &= 80\% \\ P(S|C) &= 75\% \\ P(S|C') &= 20\% \end{aligned}$$

$$\begin{aligned} P(S) &= P(S \text{ and } C) + P(S \text{ and } C') \\ P(S) &= P(C) \times P(S|C) + P(C') \times P(S|C') \\ &= 15\% + 16\% \\ &= 31\% \end{aligned}$$

$$\begin{aligned} P(C|S) &= \frac{P(S|C) \times P(C)}{P(S)} \\ &= \frac{75\% \times 20\%}{31\%} \\ &= \mathbf{48.4\%} \end{aligned}$$