Final Exam for IMSE 213, Chapters 9 and 10

Total poin
106

You may use the "cheat sheets" I created. You may make notes on the printed sides, only.
You may not use your text, HW papers or any other materials I have not handed out to you.
You may not share pencils, pens, paper, cheat sheets or calculators.
If you believe information is missing, state clearly what you are using in its place.
A "yes" or "no" answer will receive a zero unless backed up with statistical proof.
Show your work, especially equations used, limits for summations and intergrations, etc.

Assume a reasonably normal distribution unless it is obvious that it is binomial or some other distribution from the problem statement.

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No. Problem statement

1 For the following confidence intervals, tell whether to reject the null hypothesis or not:

		Ho	Confidence Interval			Reject Ho (Y or N)?
3	a)	$\sigma = 0.9$	1.01	$< \sigma <$	2.01	y
3	b)	$\sigma_1 = \sigma_2$	0.5	$< \sigma_1/\sigma_2 <$	0.9	y

2 For the following statistics, **fill in the blanks**: 5%

		H_o	H_1	Known	Distribution should be used to evaluate Ho
3	a)	$\sigma_1 = \sigma_2$	$\sigma_1 > \sigma_2$	s_1, s_2, n_1, n_2	F
3	b)	$\mu = 2$	$\mu < 2$	s, $\underline{x}$, n	t
3	c)	$\sigma = -0.5$	$\sigma <> -0.5$	σ, s, n	χ^2
3	d)	$\mu = 0$	$\mu <> 2$	σ, $\underline{x}$, n	Z

3 Express the null and alternate hypotheses for each case given below

a) You wish to be certain that the true mean value is greater than ___1.8___

1 Null: ______________ H_0: $\mu = 1.8$

1 Alternate: ______________ H_1: $\mu > 1.8$

b) A vendor claims that the standard deviation of his errors are much lower than 1.4 , the typical value. You wish to be certain he is right.

1 Null: ______________ H_0: $\sigma = 1.4$

1 Alternate: ______________ H_1: $\sigma < 1.4$

5 **4** We are trying to determine how many samples are neede in a study to determine how much severe childhood illnesses reduce adult height. It is widely believed that the reduction is ___1.5___ cm. We will not reject the null hypothesis without a certainty of at least ___95%___ . We also want a ___90%___ certainty that we would <u>not</u> accept the null hypothesis if the true reduction were actually ___2.5___ cm. The population standard deviation is ___3___ cm.

Difference = 3 | two-sided | one-sided | **Should be one-sided!!**

		two-sided	one-sided
α =	5%	Z = -1.96	-1.64
β =	10%	Z = -1.2816	-1.28

$$n \cong \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \left[\frac{(-1.96 + -1.28) \times 3.00}{1.00}\right]^2 = 94.57$$

~ 95 two-sided **Should be one-sided!!**

$$n \cong \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \left[\frac{(-1.64 + -1.28) \times 3.00}{1.00}\right]^2 = 77.07$$

~ 78 **one-sided** **Should be one-sided!!**

5 There is a contention that soccer players have a different variance in heights than the general population. The sample variance found for soccer players is given below. The American population variance is 49.00 cm^2.

	N	s^2
soccer sample	14	16.00 cm^2

5 a) Construct a 95% confidence interval for the true standard deviation based on this data.

$\sigma^2 = 49.00$ $\alpha = 0.05$

$\upsilon = 13$ $\chi^2_{\alpha/2,\upsilon} = 24.74$

$\chi^2_{1-\alpha/2,\upsilon} = 5.009$

$$P\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}}\right) = 1-\alpha$$

$$\frac{(13 \quad 16.00)}{(24.74)} < \sigma^2 < \frac{(13 \quad 16.00)}{(5.0088)}$$

$8.4 < \sigma^2 < 41.5$

$2.9 < \sigma < 6.4$

2 b) Demonstrate whether you should accept that the two populations have the same variance with a confidence of 5% ? 49.00 not in range

Reject null. Accept that soccer players have different variance than population

6a I am studying whether treating tires with a commerical product improves traction. I tested several brands of tires. I took two of each brand and sprayed one and not the other. With the data below, I want to know with ___98%___ certainty whether spraying improves traction.

Version A

2 a) State the null and alternate hypotheses:

H_o:	$\mu_d = 0$
H_1:	$\mu_d > 0$

Brand	Spray	NoSpray	diff
A	30	27	3
B	28	25	3
C	33	31	2
D	30	28	2
E	31	26	5
F	30	28	2
G	28	29	-1
H	27	29	-2
I	26	24	2
$\underline{x}$	29.22	27.44	1.78
s	2.17	2.19	2.11

5 b) Demonstrate whether I should reject the null hypothesis.

$\upsilon = 8$ $\alpha = 2\%$

$$T = \frac{1.78 - 0}{2.11 / 9^{0.5}} = \frac{1.78}{0.703} = 2.53$$

$t_\alpha = 2.45$

$T < t_\alpha$ reject null, accept H1

6b I am studying whether treating tires with a commerical product changes traction. I tested several brands of tires. I took two of each brand and sprayed one and not the other. With the data below, I want to know with 98% certainty whether spraying **changes** traction.

Version B

2 a) State the null and alternate hypotheses:

H_o: $\mu_d = 0$

H_1: $\mu_d <> 0$

Brand	Spray	NoSpray	diff
A	30	27	3
B	28	25	3
C	33	31	2
D	30	28	2
E	31	26	5
F	30	28	2
G	28	29	-1
H	27	29	-2
I	26	24	2
$\bar{x}$	29.22	27.44	1.78
s	2.17	2.19	2.11

5 b) Demonstrate whether I should reject the null hypothesis.

$$\bar{d} - \frac{t_{\alpha/2} s_d}{\sqrt{n}} < \mu_d < \bar{d} + \frac{t_{\alpha/2} s_d}{\sqrt{n}}$$

$\upsilon = 8$ $\alpha = 2\%$

$$T = \frac{1.78 - 0}{2.11 / 9^{0.5}} = \frac{1.78}{0.7027} = 2.5298$$

$t_{\alpha/2} = 2.90$

$T < t_{\alpha/2}$ accept null

5 **7** Three machines with equal production are compared for number of defects.
Can you reject the assertion that there is no difference in the 3 machines' rates of defects with a 90% confidence? Hint: same as assuming uniform distribution.

Machines	A	B	C	Total
No. caught:	13	3	5	21
Expected	7	7	7	21
(Obs-Exp)2/Exp:	5.14	2.29	0.57	8.00

$\alpha = 10\%$ $\upsilon = 2$

χ^2 = Sum of (Obs-Expected)^2/Expected

= 8.00

Must be one-sided since cannot be less than uniform.

$\chi_{\alpha,\upsilon}^2$ = 4.61

χ^2 > $\chi_{\alpha,\upsilon}^2$ Reject that equally represented.

8 I am comparing the error for two measurement devices, A and B,
It has been claimed that the variance for A is about the same as B.
Consider the test data below:

Machine A:	n = 11	s^2 = 10
Machine B:	n = 7	s^2 = 2

a) State the null and alternate hypotheses:

2 H_o: $\sigma^2 = \sigma^2$

H_1: $\sigma_A^2 <> \sigma_B^2$

5 b) Are the variances significantly different at alpha = **10%**

$$F = \frac{s_A^2}{s_B^2} = \frac{10}{2} = 5$$

$f_{5\%,10,6}$ = 4.06 < 5

F > f **reject Ho**

P(F = 5) = **0.03089** -using Excel

5 **9** A production run was sampled with the results shown below. Given the values of $\underline{x}$ and s, find the 99% two-sided confidence interval for the average output.

Sample Data

Output:	0.80	0.86	1.20	0.95	0.87	0.96	0.92	1.10	0.98

$\underline{x}$ = 0.96 s = 0.12

$\alpha/2$ = 0.005

n = 9

$t_{\alpha/2}$ = 3.355 υ = 8

$$P(\bar{X} - \frac{t_{\alpha/2}s}{\sqrt{n}} < \mu < \bar{X} + \frac{t_{\alpha/2}s}{\sqrt{n}}) = 1-\alpha$$

$$0.96 - \frac{3.36 \quad 0.12}{9.00^{0.50}} < \mu < 0.96 + \frac{3.36 \quad 0.12}{9.00^{0.50}}$$

P(0.821 < μ < 1.099) = 99%

5 **10** For the statistics below, find the ___95%___ confidence interval for the difference in samples 1 and 2: [set up equation and substitute but do not do final arithmetic].

Sample 1:	n = 10	$\underline{x}$ = 12.2	σ^2 = 1.1
Sample 2:	n = 12	$\underline{x}$ = 10.2	σ^2 = 1.0

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$\alpha = 0.05$

$$2.00 - 1.96 \left(\frac{1.10}{10} + \frac{1.00}{12} \right)^{0.5} < \mu_1 - \mu_2 < \frac{2.00}{0}$$

$$+ 1.96 \left(\frac{1.10}{10} + \frac{1.00}{12} \right)^{0.5} = 0.95$$

P(1.138 $< \mu_1 - \mu_2 <$ 2.862) = 0.95

5 **11** For the statistics below, find the 95% confidence interval for the difference in samples 1 and 2: [set up equation and substitute but do not do final arithmetic]. Assume equal variances and normal distribution.

Sample 1:	n = 8	x = 9.0	s^2 = 2.1
Sample 2:	n = 10	x = 6.0	s^2 = 1.0

$$S_p^2 = \frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{n_1 + n_2 - 2}$$

$\alpha = 0.05$

if don't pool get Sx^2 = 0.6021

$$S_p^2 = \frac{7.00 \quad 2.10 \quad + \quad 9 \quad 1.0}{8 \quad + \quad 10 \quad - \quad 2} = \frac{23.7}{16} = 1.481$$

S_p = **1.22**

Small sample, use t-tables. $t_{\alpha/2}$ = 2.12 υ = 16

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$9 - 6) - 2.12 \quad 1.22 \quad \left(\frac{1}{8} + \frac{1}{10} \right)^{0.5} < \mu_1 - \mu_2 <$$

$$3 + 2.120 \quad 1.22 \quad \left(\frac{1}{8} + \frac{1}{10} \right)^{0.5} < 0.05$$

t = 2.47

P(1.776 $< \mu_1 - \mu_2 <$ 4.224) = 0.05 don't show

5 **12** We are planning a study with the expected values below:

Expected: $\sigma = 2.4$ n = 16 $\mu = 8$

Ho: $\mu =$ 8

H_1: μ <> 8

If $\alpha =$ 5% what is the beta value if the true value could be ___9.5___ ?

Z= x - μ/(σ/n^.5) = -1.96

$x_1 = Z_1\ \sigma/n^{0.5} + \mu =$ 6.82402 $x_2 =$ 9.176

P(Type **II**) = P($\underline{X}$< 9.176) - P($\underline{X}$<6.82402) given that μ = **9.5**

$$Z_1 = \frac{\underline{x}_1 - \mu_0}{\sigma/n^{0.5}} = \frac{6.82402 - 9.5}{2.4\ /\ 16\ \hat{}0.5} = -4.46$$

$$Z_2 = \frac{\underline{x}_2 - \mu_0}{\sigma/n^{0.5}} = \frac{9.17598 - 9.5}{2.4\ /\ 16\ \hat{}0.5} = -0.54$$

$P(t<T_1) =$ 0.00023 $P(t<T_2) =$ 0.2985

P(Type **II**) = 0.29855 - 0.0002 = **0.30**

5 **13** A random sample produced the following proportions of preference for vanilla ice-cream:

Proportion that preferred vanilla over other flavors

33	of	50	Age less than 30
22	of	40	Age greater than 30

Compute the 90% confidence interval for the difference between these two groups.

Solution: p_{fac} = 0.660 p_{stu} = 0.550 $Z_{\alpha/2}$ = 1.645

p_{diff} = 0.110

$$(\hat{p}_1 - \hat{p}_2) - Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \left(\frac{0.224}{50} + \frac{0.248}{40}\right)^{0.5} = 0.103$$

(0.660 0.340; 0.550 0.450)

P(0.110 - 1.645 0.103 < p < 0.110 + 1.64 0.103) = 1.00

-0.060 < p_1-p_2 < 0.280

14 You are testing to see if car owners under-inflate their tires (i.e., lower than __32__ psi. Based on the data below:

Observed Pressures					$\bar{x}$	s
26	28	35	38	31	29.87	5.00
26	32	23	34	27		
22	37	34	29	26		

a) State the null and alternate hypotheses:

2

H_o: $\mu = 32$

H_1: $\mu < 32$

5 b) Should you accept or reject at __5%__ ?

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} =$$

29.867	-	32	=	-2.1333	=	-1.653
4.9981	/	15 ^.5		1.2905		

P(t) = 0.0603 $\upsilon = 14$

$t_\alpha = -1.7613 < T$ **Cannot reject**

5 **15** A production run was sampled with the results shown below. Given the values of x and s, find the 99% two-sided PREDICTION interval for the NEXT VALUE TO APPEAR.

Sample Data

Output:	0.80	0.86	1.20	0.95	0.87	0.96	0.92	1.10	0.98

x = 0.96 s = 0.12

$\alpha/2$ = 0.005

n = 9

$t_{\alpha/2}$ = 3.355 υ = 8

$$P(\bar{X} - t_{\alpha/2} s\sqrt{1+1/n} < x_o < \bar{X} + t_{\alpha/2} s\sqrt{1+1/n}) = 1-\alpha$$

0.96 - 3.36 0.12 1.11 $^{0.50}$ < μ < 0.96 + 3.36 0.12 1.11 $^{0.50}$

P(0.521 < μ < 1.399) = 99%