

Practice First Exam for IENG 213

You may use your text and any "cheat sheets" you created.
 You may not use your HW papers or the statistical functions on a calculator
 You may not share texts, cheat sheets, or calculators.
 Show your work, especially equations used, limits for summations and intergrations, etc.

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- | Pts. | No. | Problem statement | Comment: This is based on one of my first tests. |
|------|-----|---|---|
| | 1 | What is the probability of drawing 3 aces in a row from a deck of 52 if you: | |
| 2 | a) | Replace the card drawn and re-shuffle the deck each time | $= (4/52)^3$ comment: too easy |
| 2 | b) | Discard each card as it is drawn and re-shuffle the rest of the deck before drawing again | $= (4/52)(3/51)(2/50)$ |
| 1 | 2 | A card is drawn from a ordinary 52-card deck. What is the probability that it is a 3, given that it is a face card? _____ | $= 0$ comment: too easy |
| 3 | 3 | Given the following data, compute the median: Data: -4 1 2 5 Median: _____ | comment: too easy $1.500 = (\text{two middle})/2$ |
| 2 | 4 | What is the mean and the standard deviation of the data above? Mean: _____ | 1.000 comment: too easy |
| 3 | | Std. Dev.: _____ | 3.742 |
| 3 | 5 | Given that $P(A) = 0.6$ and $P(B) = 0.2$ and A and B are independent. | |
| 3 | a) | $P(A \cup B) =$ _____ | 0.680 |
| 3 | b) | $P(A \cap B) =$ _____ | 0.120 comment: should be word problem |
| 6 | 6 | Three computer brands are each sold with 5 different RAM configurations and 2 different harddisk sizes. How many choices do I have? | 30 comment: too easy |

- 7 7 Find the number of ways 5 men can be assigned to drive 4 cars if no driver can drive more than one car. comment: too easy

if indistinguishable cars, then:
$$N = \frac{5!}{4!(5-4)!} = \frac{5!}{4!} = 5$$

if distinguishable cars, then:
$$N = \frac{5!}{(5-4)!} = 5! = 120$$

- 7 8 Find P(A|B) given the following: Comment: Replace with total probability problem

P(A)= 0.10 P(B|A) = 0.80

P(B)= 0.20 P(B|A') = 0.06

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Thus,
$$P(A \cap B) = 0.80 * 0.10 = 0.08$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.20} = \boxed{0.40}$$

Or:
$$P(A|B) = \frac{P(B|A)*P(A)}{P(B)} = \frac{0.08}{0.20} = \boxed{0.40}$$

- 15 9 For the following continuous frequency distribution, find P(0 < x < 0.4) = _____

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$P(0 < x < 0.4) = \int_0^{0.4} 3x^2 dx = x^3 \Big|_0^{0.4} = 0.064 - 0.000 = 0.064$$

10 For the function to the right: $f(x, y) = 4xy$, $0 \leq x \leq 1$, $0 \leq y \leq 1$
 $= 0$, otherwise

7 a) Find the marginal distribution of x

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy = 0 + \int_0^1 4xy dy = 2xy^2 \Big|_0^1 = 2x - 0 = 2x$$

7 b) Find the marginal distribution y

$$h(y) = \int_{-\infty}^{\infty} f(x, y) dx = 0 + \int_0^1 4xy dx = 2x^2 y \Big|_0^1 = 2y - 0 = 2y$$

7 c) Find the conditional probability distribution of x given y :

$$f(x | y) = \frac{f(x, y)}{h(y)} = \frac{4xy}{2y} = 2x \quad 0 < x < 1, 0 < y < 1$$

7 d) Find the probability X is between 0 and 0.1

$$P(0 < x < 0.1) = \int_0^{0.1} g(x) dx = \int_0^{0.1} 2x dx = x^2 \Big|_0^{0.1} = 0.01$$

e) Find the probability X < 1

100%

e) Determine whether $f(x,y)$ is a possible distribution defined over all possible values

sum of $P = 1$

all $P > 0$

$$P(\text{range}) = 1 = \int_{-\infty}^{\infty} f(x, y) dx dy = 0 + \int_0^1 \int_0^1 4xy dx dy$$

$$= \int_0^1 2x^2 y dy \Big|_0^1 = \int_0^1 2y dy = y^2 \Big|_0^1 = 1$$

f) Are X and Y independent?

Is $f(x,y) = g(x) h(y)$ for all x and y?

$4xy = 2x \cdot 2y = 4xy$ **yes**

11 Given the joint probability distribution below:

| | | | | | | | | |
|---------------|----------|-------------|-------------|-------------|-------------|--------------------|------|------|
| f(x,y) | x | 1 | 2 | 3 | h(y) | g(x) * h(y) | | |
| | 1 | 0.10 | 0.05 | 0.05 | 0.20 | 0.03 | 0.07 | 0.03 |
| y | 2 | 0.05 | 0.10 | 0.35 | 0.50 | 0.08 | 0.18 | 0.18 |
| | 3 | 0.00 | 0.20 | 0.10 | 0.30 | 0.05 | 0.11 | 0.05 |
| | | g(x) | 0.15 | 0.35 | 0.50 | | | |

5 a) List the marginal distribution of X

see above

5 b) List the marginal distribution of Y (see above)

see above

5 c) Find $P(Y=2 | X=3) = \frac{f(3,2)}{g(3)} = \frac{0.35}{0.50} = 0.7$

d) Are X and Y independent ?

No, since $f(x,y) \neq g(x) h(y)$ for all x and y

More typical questions

- o 1 If there are 40 good and 10 defective fuses, and 3 fuses are selected, what is the probability that all of them will be good?

Not independent since choices reduced by previous outcomes:

$$P = 40/50 \times 39/49 \times 38/48 = 0.800 \times 0.796 \times 0.792 = 0.504$$

- o 2 Given the following table listing the ethnicity of a college class, fill in the frequency distribution table to the right of the data.

| | | | | | | |
|-----|--------|------------------------|------|-----|------|------|
| No. | Ethnic | Frequency Distribution | | | | |
| 3 | B | x | B | H | AI | C |
| 4 | H | f(x) | 0.15 | 0.2 | 0.05 | 0.60 |
| 1 | AI | | | | | |
| 12 | C | | | | | |
| 20 | Total | | | | | |

- o 3 Is there anything that appears wrong about the following distribution?

| | | | | | |
|------|------|------|------|------|------|
| x | 0 | 1 | 2 | 3 | 4 |
| f(x) | 0.28 | 0.22 | 0.33 | 0.17 | 0.11 |

Answer: 1.11 Exceeds unity

20 4 For the continuous random variable x , where: $f(x) = \begin{cases} (2 - 3x^2), & 0 < x < 1 \\ 0 & \text{all else} \end{cases}$

a) Find the mean of the continuous random variable x

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = 0 + \int_0^1 x (2 - 3x^2) dx = x^2 - \frac{3x^4}{4} \Big|_0^1 = 1/4$$

b) Find the variance of the continuous random variable x [Set up, only]

$$\sigma_x^2 = E(X^2) - \mu_x^2 \quad \mu_x = 1/4$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = 0 + \int_0^1 x^2 (2 - 3x^2) dx$$

c) Find the probability that $0 < x < 0.2$:

$$P(0 < x < 0.2) = \int_0^{0.2} (2 - 3x^2) dx = 2x - x^3 \Big|_0^{0.2} = 0.4 - 0.20^3 = \mathbf{0.39}$$

5 Given the following, find the mean of Z and the variance of Z

$$z = 2x - y + 4 \quad \mu_x = 1 \quad \sigma_x^2 = 1$$

$$\mu_y = 2 \quad \sigma_y^2 = 2$$

$$\sigma_{xy} = -3$$

$$\begin{aligned} \mu_z &= a \mu_x + b \mu_y + c \\ &= 2 \cdot 1 + (-1) \cdot 2 + 4 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \sigma_z^2 &= a^2 \sigma_x^2 + b^2 \sigma_y^2 + 2ab \sigma_{xy} \\ &= 4 \cdot 1 + 1 \cdot 2 + 2 \cdot 2 \cdot (-1) \\ &= 18 \end{aligned}$$