96 # Practice Second Exam for IENG 213 originally 30 Oct 02

Covers Chapters 5, 6, 8

Pts **No.** Problem statement

1 Of __80__ students entering engineering, __20__ dropped out the first year.
Of the next __15__ entering students, find the probability (assuming independence):

5 a) Fewer than __3__ drop out

n = 15 p= 0.25 0.12683 0.027114
x = 2 interpolation= 0.0969177

P(x< 3 | p = 0.25) = 0.2361

5 b) More than __7__ drop out

P(x> 7 | p = 0.25) = 1 - 0.983 = **0.017**

3 c) How many of the __15__ entering students would you expect to drop out based on past experience? [The answer could be a non-integer].

$\mu = n\,p =$ 15 * 0.25 = **3.75**

3 d) What is the standard deviation of the mean number expected in part c above?

$\sigma^2 = npq =$ 15 * 0.25 * 0.75 = **2.8125**

8 **2** You and 11 friends went to the same party. Later 15 people from the party (not counting you gathered outside on the lawn. What is probability that 6 of those on the lawn were your friends if there were 25 people at the party other than you?

Set up the problem and substitute values for variables without doing the final computations.

x= 6 N= 25 n= 15 k= 11

$$h(x;N,n,k)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

Hypergeometric

$$h(x;25,15,10)=\frac{\binom{11}{x}\binom{25-11}{15-x}}{\binom{25}{15}}=\frac{\binom{11}{6}\binom{25-11}{15-6}}{\binom{25}{15}}=\frac{\binom{11}{6}\binom{15}{9}}{\binom{25}{15}}$$

8 **5** A given component has a 10% defective rate. For a sample of 100 items selected at random, find the probability that you will find less tha 8 defectives. Assume independence

Use normal approximation.

binomial-normal approx

n= 100 p= 10% x= 7

μ = np = 10.0

σ^2 = npq = 9.0 σ = 3

x_1 = 7.5

$$Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{-2.50}{3.00} = -0.8333$$

$P(Z_1)$ = **0.2023**

8 **6** Length of time to serve one person is a Poisson process with a mean of
5 minutes. What is the probability that the next person is served in less than
4 minutes?

exponential

$\beta =$ 5 minutes

$$f(x) = \begin{cases} \dfrac{e^{-x/\beta}}{\beta} & x > 0 \\ 0 & elsewhere \end{cases}$$

$$P(X < 4) = \int_0^4 \frac{e^{-x/\beta}}{\beta} dx = \frac{1}{5}\int_0^4 e^{-x/5} dx$$

$$= -(5/1)(1/5)\, e^{-x/5} \Big|_0^4 = 1 - e^{-4/5}$$

= **0.5507**

5 b) What is the standard deviation of the number served in 5 minutes?

$\sigma^2 = \alpha\beta^2 = 25$

$\sigma = 5$

8 c) What is the probability that 2 out of the next 3 people selected at random will be served in less than 4 minutes ?

binomial: P = b(2 3 0.5507) = 0.40876

= B(2 3 0.5507

- B(1 3 0.5507)

7 Soft drink volumes have the population mean and standard deviation shown below.
A sample of 9 are checked. If the mean of the sample is withi 2 standard deviations, it is assumed to be okay.

μ =	340	ml	σ =	24	ml
$\underline{x}$ =	320	ml	n =	9	

5 a) According to their criteria, does the machine need adjustment?

$$Z = \frac{X_{avg} - \mu}{\sigma / n^{0.5}} = 2.5 > \underline{\mathbf{2}}$$

NOT okay

3 b) Assuming the population values are correct, what is the probability that the next soft drink dispensed will be less tha 316

Z = -1

P(X < 316) = **0.16**

3 **8** If the distribution of a set of data is perfectly normal, what will the $q_{1,0}$ vs Quantile plot look like?

linear

8 **9** Find the probability that a random sample of ___22___ observations from a normal population with a sample variance of ___5___ has a population variance between: ___4.00___ and ___6.00___

$\upsilon = 21$

$\chi_1^2 =$ 26.25	chi-square	$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$

$\chi_2^2 =$ 17.50

$P(\chi^2 >$ 26.25) = 0.20

$P(\chi^2 >$ 17.50) = 0.68	Table A.5 column headingwrong

P(range) = **0.48**

should be: 0.75 0.70 0.50
not: 0.75 0.75 0.50

8 **10** Consider the following measurements, show whether the population variance of population 1 is equal to population 2? Use α = ___0.05___ in deciding significance.

	Observed values						S^2	X_{avg}
Pop1	8456	7800	8310	7700			138782	8067
Pop2	8010	7680	8013	8020	7920	7780	20476	7904

F = 6.7777	$f_{3,5}$ at α=0.05 = 5.4095

$P(\sigma / \sigma = 1)$ = **0.033**

8 **11** A machine breaks down 2 times per month on the average. What is the probability it will break down 3 times this month? [Assume breakdowns are independent and cannot occur simultaneously in a Poisson process].

P(x = 3) $= p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} = \frac{e^{-2}(2)^3}{3!}$

= $\frac{0.1353 \quad 8}{6}$

= **0.1804** 0.1804 --> excell

8 **12** If the probability of your lawnmower starting on a given pull is independent of the number times you have tried, what is the probability it will finally start <u>before</u> tl 3 rd pull pull given that the probability of starting on any given pull is 20% ?

$$P(x<3) = \sum_{x=1}^{2} g(x;p) = \sum_{x=1}^{2} g(x;0.2)$$

x = 1

g(1;p) = p q^{x-1} = 20% * 80% ^ 0 = 20%

x = 2

g(1;p) = p q^{x-1} = 20% * 80% ^ 1 = 16%

P(x < 3) = 20% + 16% = **36%**