No. Chapter 3: Section B

1 Determine the value of c so that each of the following function represents a legitimate joint probability distribution of the random variables X and Y:

a) for $f(x,y) = cxy, \quad for\ x = 1,2,3 \quad y = 1,2,3$

b) f(x,y) = c|x-y|, for x= -2,0,2 y= -2,3 Part b not assigned.

$$\sum_{y=-2,3}\sum_{x=-2,0,2} c|x-y| = c\sum_{y=-2,3}\sum_{x=-2,0,2}|x-y|$$

$$= c\sum_{y=-2,3}|-2-y|+|0-y|+|2-y|$$

$$\left(|-2+2|+|0+2|+|2+2|+|-2-3|+|0-3|+|2-3|\right)c$$

$$\left(0+2+4+5+3+1\right)c = 15c = 1$$

$$c = 1/15$$

2 A sack contains 3 oranges, 2 apples, and 3 bananas. Take 4 at random.

a) List the joint probability distribution of X and Y (the first steps are done for you)

$N_{total} = \frac{N}{r!(N-r)!} =$ N = 8, r = 4

$X = N_{oranges}$ $Y = N_{apples}$ $4 - x - y = N_{bananas}$ Since r = 4, then X+Y ≤ 4

$$f(x,y) = \frac{\binom{3}{x}\binom{3}{y}\binom{3}{4-x-y}}{N_{total}}$$

(oranges, apples, bananas)

$$f(x,y) = \frac{\dfrac{3!}{x!(3-x)!}\;\dfrac{2!}{y!(2-y)!}\;\dfrac{3!}{(4-x-y)!(-1+x+y)}}{N_{total}}$$

x = 0,1,2,3 X+Y ≤ 4

y = 0,1,2 Other = 4 - (x+y)

$$f(0,1) = \frac{\frac{72}{6 \quad 1 \quad 6}}{70} = \frac{2}{70}$$ r = x 0, y 1, other 3

$$f(1,0) = \frac{\frac{72}{2 \quad 2 \quad 6}}{70} = \frac{3}{70}$$ r = 1 0 3

$$f(2,0) = \frac{\frac{72}{2 \quad 2 \quad 2}}{70} = \frac{9}{70}$$ r = 2 0 2

$$f(3,1) = \frac{\frac{72}{6 \quad 1 \quad 6}}{70} = \frac{2}{70}$$ r = 3 1 0

f(0,2) =

f(1,2) =

f(2,2) =

f(1,1) = $\frac{72}{\quad}$ = $\underline{18}$ r = 1 1 2

$$\frac{}{\frac{2 \quad 1 \quad 2}{70}} \qquad 70$$

$$f(2,1) = \frac{72}{\frac{2 \quad 1 \quad 2}{70}} = \frac{18}{70} \qquad r = 2 \quad 1 \quad 1$$

$$f(3,0) = \frac{72}{\frac{6 \quad 2 \quad 2}{70}} = \frac{3}{70} \qquad r = 3 \quad 0 \quad 1$$

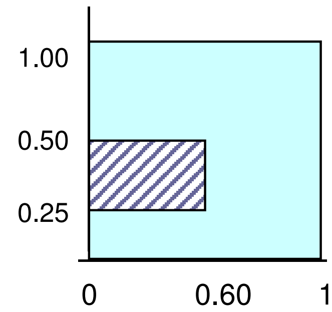
b) Using the values computed above, fill in the table below:

	X: 0	1	2	3
y 0		3/70	9/70	3/70
y 1		18/70	18/70	2/70
y 2		9/70	3/70	0/70

c) Determine P(x,y| x +y ≤ 2) **0.50**

3 X= reaction time Y= length of life of electronic components
Continuous joint probability distribution:

$$f(x,y) = \begin{cases} 4xy & 0 < x < 1,\ 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

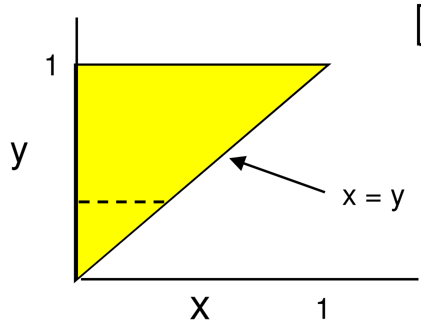


a) Compute: P(0 < X < 0.6 and 0.25 < Y < 0.5)

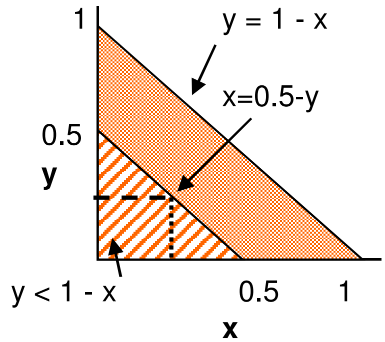
6.8%

b) Compute: P(X < Y)

1/2



4 Candy boxes are mixture of creams, toffees, and cordials.
Wt of box = 1 kg
x = creams y = toffees
Part A is not assigned. Parts B and C are assigned.



$$f(x,y) = \begin{vmatrix} 24xy & 0 \le x \le 1,\ 0 \le y \le 1 \\ & x + y \le 1 \\ 0 & elsewhere \end{vmatrix}$$

a) P(cordials >0.5 Wt) ?

Hints: If x+y < 1 then one upper limit of x = 1 - y
However, since the upper limit of x + y = 0.5, then upper limit of x = 0.5 - y
Can state in terms of y instead of x, if desired. Just be sure to apply it to the first integration, not the second.
The outside (i.e., second) integration limits are not affected since accounted for already

$$p(x+y \le 0.5) = \int_0^{0.5} \int_0^{0.5-y} 24xy\, dxdy$$

$$= \int_0^{0.5} 12x^2 y\, dy \Bigg|_0^{0.5-y} = 12\int_0^{0.5} (0.5-y)^2 y\, dy$$

$$= 12\int_0^{0.5} (\frac{y}{4} - y^2 + y^3)\, dy = 12(\frac{y^2}{8} - \frac{y^3}{3} + \frac{y^4}{4})\Bigg|_0^{0.5}$$

$$= 12y^2(\frac{1}{8} - \frac{y}{3} + \frac{y^2}{4})\Bigg|_0^{0.5}$$

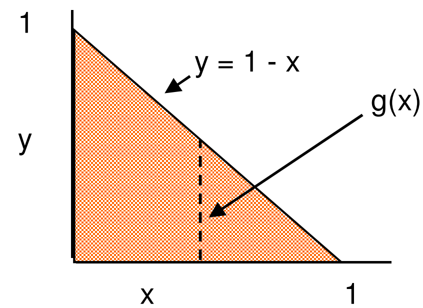
$$= 12(0.5^2)[1/8 - 0.5/3 + 0.5^2/4] - 0$$

$$= 3[1/8 - 0.5/3 + 0.25/4] =$$

$$= \quad 0.0625$$

b) Find g(x), the marginal distribution of creams
This is assigned

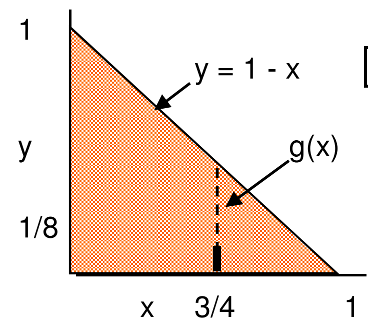
$$f(x,y)=\begin{cases}24xy & 0\le x\le 1,\ 0\le y\le 1 \\ & x+y\le 1 \\ 0 & elsewhere\end{cases}$$



Hint: note that x+y <1 sets a restriction on the limits for each of the marginal distributions.

$12x(1-x)^2$

c) Find the probability that the weight of toffee <1/8 kg given that the weight of cream = 3/4 kg
This is assigned

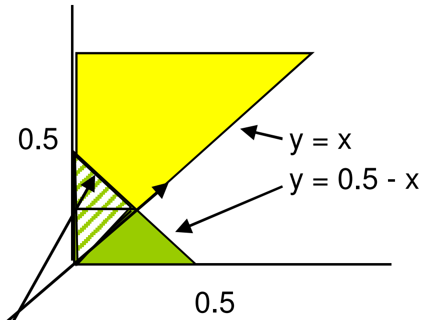


0.25

5 Find P(X+Y) < 1/2

X= diameter of cable Y= diameter of mold

Not assigned



$$f(x,y) = \begin{cases} 1/y & 0 < x < y < 1 \\ 0 & elsewhere \end{cases}$$

If x + y < 0.5:

In the range of 0 < y < 1/4, the sum of any x and y will be less than 1/2, therefore the value of x can be as high as y, 0 < x < y

In the range of 1/4 < y < 1/2, the sum of any x and y will be greater than 1/2 if x=y, therefore the value of x can must be limited to 0 < x < 1/2-y

0 < x < y 0 < y < 1/2

$$(X+Y>0.5) = 1 - P(X+Y<0.5)$$

$$= 1 - \int_0^{1/4}\int_0^{y} \frac{1}{y}\,dx\,dy - \int_{1/4}^{1/2}\int_0^{1/2-y} \frac{1}{y}\,dx\,dy$$

$$= 1 - \int_0^{1/4} \frac{1}{y}x\,dy \Big|_0^{y} - \int_{1/4}^{1/2} \frac{1}{y}x\,dy \Big|_0^{1/2-y}$$

$$= 1 - \int_0^{1/4} \left(\frac{1}{y}\right) y\,dy - \int_{1/4}^{1/2} \left(\frac{1}{y}\right)(1/2-y)\,dy$$

$$= 1 - \int_0^{1/4} dy - \int_{1/4}^{1/2} \left[\left(\frac{1}{2y}\right) - 1\right] dy$$

$$= 1 - y\Big|_0^{1/4} - (1/2)\ln(y)\Big|_{1/4}^{1/2} - (-1)y\Big|_{1/4}^{1/2}$$

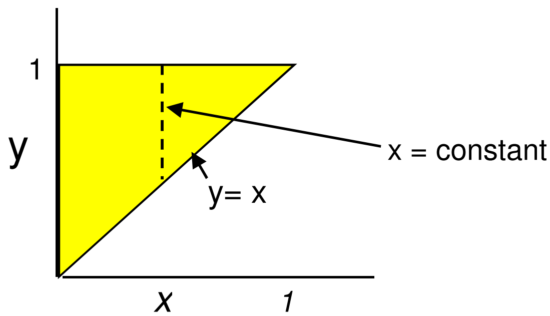
$$= 1 - (1/4 - 0) - (-1)(1/2 - 1/4) - 0.5[\ln(1/2) - \ln(1/4)]$$

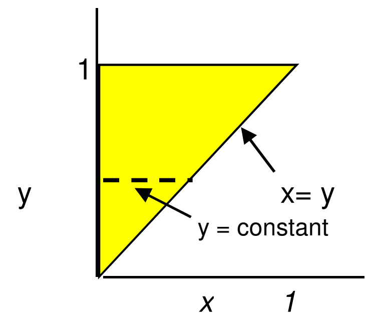
0.65

6 The amount of kerosene in a tank at the beginning of any day is a random amount, Y, from which a random amount X is sold that day. The tank is not re-supplied during the day, so X ≤ Y. Assume the continuous joint probability density function is:

$$f(x,y) = \begin{cases} 2 & 0 < x < y,\ 0 < y < 1 \\ 0 & \textit{elsewhere} \end{cases}$$

X = amount sold
Y = amount stored at the beginning of the day
X ≤ Y (can't sell more than have)





a) Are X and Y independent? Hint: for each marginal distribution, x < y affects limits of integration.

b) Find $P(0.25 < x < 0.6 \mid y = 0.5)$: 70%

7 Given the table of discrete joint probabilities, $f(x,y)$, where:

x = Number of times machine will fail in a day

y = number of times technician called as an emergency

		x		
	0	0	1	2
y	1	0.08	0.09	0.10
	2	0.07	0.10	0.10
	3	0.06	0.20	0.20

a) Evaluate the marginal distribution of X

b) Evaluate the marginal distribution of Y

c) Find $P(Y=3|\ X=0)$ =

8 X and Y have the following joint probability distribution:

		X	
		2	4
y	2.00	0.10	0.15
	3.00	0.17	0.27
	4.00	0.16	0.15

a) Evaluate the marginal distribution of X (see above)

b) Evaluate the marginal distribution of Y (see above)

c) Compute P(x = 2 | y = 3) =

9 Given the following joint prob function, find: P(1 < y < 3 | x = 2)

$$f(x,y) = \begin{cases} \dfrac{6-x-y}{8} & 0<x<2,\ 2<y<4 \\ 0 & elsewhere \end{cases}$$

10 Given the discrete distribution below:

For all other combinations of x and y, f(x,y) = 0

		x = 1	x = 2	x = 3
y	1	5%	7%	10%
	2	9%	21%	21%
	3	5%	12%	10%

a) Determine whether it can be a legitimate probability distribution:

b) Find P(x = 3 and y = 1) =

c) List the marginal distribution of x:

d) Find P(x =3 | y = 1) =

e) Determine whether X and Y are independent:

11 Determine whether the two random variables in the table below are independent.

f(x,y)	x	1	2
	1	0.10	0.20
y	3	0.25	0.30
	5	0.10	0.05

g(x)

12 The joint density function of the random variables X and Y is:

$f(x,y) = 6x$ for $0<x<1$ and $0<y<1-x$
$= 0$ elsewhere

Show that x and y are NOT independent

13 For joint prob distr below:

Not assigned.

$$f(x,y,z) = \begin{vmatrix} kxy^2z & 0<x<1,\ 0<y<1, \\ & 0<z<1 \\ 0 & elsewhere \end{vmatrix}$$

a) Find k

$$1 = \int_0^2 \int_0^1 \int_0^1 kxy^2z\ dxdydz = \qquad = \frac{k}{2}\int_0^2 \int_0^1 x^2y^2z\ dydz \Big|_0^1$$

$$= \frac{k}{2}\int_0^2 \int_0^1 yz^2\ dydz = \frac{k}{6}\int_0^2 y^3z\ dx = \Big|_0^1$$

$$= \frac{k}{6}\int_0^2 z\ dz = \left(\frac{k}{12}\right)z^2 \Big|_0^2 = 4k/12 = 1$$

$$k = 3$$

b) Find P(X < 1/4, Y > 1/2 , 1 < Z < 2)
Not assigned.

$$f(x,y,z)=\begin{cases} kxy^2z & 0<x<1,\ 0<y<1,\ 0<z<2 \\ 0 & elsewhere \end{cases}$$

$$P(X<1/4, Y>1/2, 1<Z<2)$$

$$=3\int_1^2\int_{1/2}^1\int_0^{1/4} xy^2z\,dx\,dy\,dz$$

$$=\frac{3}{2}\int_1^2\int_{1/2}^1\int_0^{1/4} x^2y^2z\,dy\,dz\Big|_0^{1/4}=\frac{3}{2}\int_1^2\int_{1/2}^1 (1/4)^2y^2z-0\,dy\,dz$$

$$=\frac{3}{32}\int_1^2\int_{1/2}^1 y^2z-0\,dy\,dz=\frac{1}{32}\int_1^2 y^3z\,dz\,{}_{1/2}^{1}$$

$$=\frac{1}{32}\int_1^2\left[(1)^3-(1/2)^3\right]z\,dz=\frac{7}{8*32}\int_1^2 z\,dz=\left(\frac{7}{8*32}\right)z^2\,dz\Big|_1^2$$

$$=\left(\frac{7}{8*32}\right)(4-1)=\left(\frac{21}{16*32}\right)$$ = 21/512 = 0.04

14 Die is rolled twice. X= Number of 4s and Y= Number of 5s obtained in two rolls.

Joint probability Not assigned

$N_{total} = 6 * 6 = 36$

$0 < x + y \leq 2$ since can't roll more than 2 of same value

Possible outcomes:

S = {1,1; 1,2; 1,3; 1,4; 1,5; 1,6; 2,1; 2,2; 2,3; 2,4; 2,5; 2,6;
3,1; 3,2; 3,3; 3,4; 3,5; 3,6; 4,1; 4,2; 4,3; 4,4; 4,5; 4,6;
5,1; 5,2; 5,3; 5,4; 5,5; 5,6; 6,1; 6,2; 6,3; 6,4; 6,5; 6,6)

f(x,y)

f(0,0) =	16/36	f(0,2) =	1/36	f(0,0) = (6-2)(6-2)
f(0,1) =	8/36	f(2,0) =	1/36	
f(1,0) =	8/36	f(1,1) =	2/36	

Region of A:

P(x,y|[2X + Y]<3) = f(0,0) + f(0,1) + f(0,2) + f(1,0)

= 16/36+8/36+1/36+8/36 = 33/36

= 0.9167 =11/12

10 Randomly selected three cards without replacement from 12 face cards.
Let X= Number of kings and Y=Number of jacks.
Not assigned

a) Define: f(x,y) x= 0,1,2,3 y= 0,1,2,3
$0 \le x + y \le 3$

$$f(x,y) = \frac{\binom{4}{x}\binom{4}{y}}{\binom{12}{3}} = \frac{\frac{4!}{x!(4-x)!} \cdot \frac{4!}{y!(4-y)!}}{\frac{12!}{3!(12-3)!}}$$

Example:

$$f(1,2) = \frac{\frac{4!}{1!(4-1)!} \cdot \frac{4!}{2!(4-2)!}}{220}$$

$$= \frac{4 \cdot 6}{220} = \frac{6}{55}$$

Table Created from Above

f(x,y)		x = 0	1	2	3
y	0	1/55	6/55	6/55	1/55
	1	6/55	16/55	6/55	
	2	6/55	6/55		
	3	1/55			

b) Compute P(X + Y > 2) from the table above:

Not assigned

			x		
f(x,y)	0	0	1	2	3
	0	1/55	6/55	6/55	1/55
y	1	6/55	16/55	6/55	
	2	6/55	6/55		
	3	1/55			