No. Problem statement

1 Classify as random or discrete:

	<u>discrete</u>	<u>continuous</u>
number of auto accidents per year		
time to play 18 holes of golf		
amount of milk		
number of eggs		
number of building permits		
weight of grain		

2 Let W = number of heads minus the number of tails in 3 tosses of a coin.

List the sample space, S =

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3 Determine the value, c, that would have to be true for each of the following functions to be a legitimate discrete probability distribution

a) $f(x) = c(x^2+4)$ $X = 0,1,2,3$

b) Find the probability that X = 0, 1, and 2, respectively:

For your review; not assigned

$$f(x) = c\binom{2}{x}\binom{3}{3-x} = \left[\frac{2!}{x!(2-x)!}\right]\left[\frac{3!}{(3-x)!(3-(3-x))!}\right], \quad x = 0,1,2$$

$$= \left[\frac{2!}{x!(2-x)!}\right]\left[\frac{3!}{(3-x)!(x))!}\right]$$

$$\sum_{x=0}^{2} f(x) = c\left[\frac{2!}{0!(2-0)!}\right]\left[\frac{3!}{(0)!(3-0)!}\right] + c\left[\frac{2!}{1!(2-1)!}\right]\left[\frac{3!}{(1)!(3-1)!}\right]$$

$$+ c\left[\frac{2!}{2!(2-1)!}\right]\left[\frac{3!}{(3)!(3-3)!}\right] = c\left[1+(2)(1)+2(3)+1(1)\right] = 10c$$

$$\sum_{x=0}^{2} f(x) = 1 = 10c$$

$$c = 1/10$$

4 Assuming that the function below is a legitimate continuous probability distribution, find: P(X > 100)

0.25

$$f(x) = \begin{cases} \dfrac{20{,}000}{(x+100)^3} & X > 0 \\ 0 & elsewhere \end{cases}$$

Hint: let u = x+100

5 Given that the function below is a legitimate probability distribution,
X= decades until failure for a vacuum cleaners

$$f(x) = \begin{vmatrix} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & elsewhere \end{vmatrix}$$

a) Find probability that a vacuum will last MORE than 1 decade:

b) Between 0.1 and 0.7 decades: 0.24

6 Respondents to solicitation has continuous density function:

$$f(x) = \begin{cases} \dfrac{2(x+2)}{5} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

a) Show that P(0 < X < 1) = 1

b) Find the probability that more than 25% but fewer than 60% of the people contacted will respond.

0.34

7 The probability distribution of X, imperfections per 10m is:

x	0	1	2	3	4
f(x)	0.10	0.20	0.40	0.20	0.10

Construct a table for the cumulative distribution

x	< 0	< 1	< 2	< 3	< 4
F(x)					

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8 The waiting time between successive speeders at an intersection has the cumulative probability density distribution:

$F(x) = 0$ for $X \le 0$ $F(x) = 1 - e^{-8x}$ for $x > 0$ hours

a) Find the probability of waiting less than 0.15 hours for the next speeder using the cumulative probability density distribution

0.699

b) Find the probability density function based on the cumulative distribution above:

c) Find the probability of waiting less than 0.1 hours for the next speeder using the probability density distribution

0.551

9 Find the cumulative distribution of the random variable, X, below:

x	0	1	2
f(x)	2/11	5/11	4/11
F(x)			

a) Find P(X=1) =

b) Find $P(0 < X \le 2)$=

[note that the boundaries are important]

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10 A continuous random variable, X, that can assume values between x = 1 and x = 3 have a density function given by:

$f(x) = 0.5 \qquad 1 < x < 3$
$f(x) = 0 \qquad \text{all else}$

a) Show that the area under the curve = 1.

b) Find $P(1.5 < X < 3)$

0.75

c) Find $P(X \leq 1.8)$

11 Given the continuous random variable, x, and $f(x) = (2/27)(1 + x) \qquad 2 < x < 5$
$= 0 \qquad \text{elsewhere}$

a) Find $P(X < 3)$

0.259

b) Find $P(2 < X < 3)$

0.259

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12 Consider the density function:

$$f(x) = \begin{cases} k\sqrt{x} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

a) Evaluate k:

b) Find F(x) and use it to evaluate P(0.4 < X < 0.75)

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13 An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production dat in the past, it has been determined that the particle size in microns distribution is characterized by:

$$f(x) = \begin{cases} 3x^{-4} & x > 1 \\ 0 & Elsewhere \end{cases}$$

a) Verify that this is a valid distribution function.

b) What is the probability that a random particle from the mfg fuel exceeds 5 microns ? 0.8%

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14 Based on extensive testing, it is determined by the manufacturer of a washing machine that the time Y (in years) before a major repair is required is characterized by the probability density function:

$$f(x) = \begin{cases} \dfrac{e^{-y/4}}{4} & y \geq 0 \\ 0 & Elsewhere \end{cases}$$

a) Find P(Y > 10) 8.2%

b) What is the probability that the first major repair occurs after year = 2 ?

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15 3 cards drawn without replacement. Find prob distribution for number of spades

x = number of spades drawn Extra problem

S = {0, 1, 2, 3}

spades not spades

Ngroups of 3 = $\frac{52!}{(52-3)!(3!)} = \frac{52*51*49!}{49!3!} =$ 442

$N_{spades} = \frac{3!}{x!(3-x)!} \times \frac{(52-x)!}{(52-x-3+x)!(3-x)!} = \frac{3!}{x!(3-x)!} \times \frac{(52-x)!}{(49)!(3-x)!}$

x = 0 N =

16 From a box of 4 dimes and 2 nickels, 3 coins are selected without replacement. Find the probability distribution. For the summed total, T, of the 3 coins, express the probability distribution graphically as a probability histogram.

Extra problem

NoDimes	T
3	30
2	25
1	20

TotalCombos = $\frac{6!}{3!(6-3)!} = \frac{6*5*4*3!}{3!3!} =$ 20

$N_{20} = \frac{2!}{2!(2-2)!} * \frac{4!}{1!(4-1)!} =$ 4 p(T=20) = 4 / 20 = 0.20

$N_{25} = \frac{2!}{1!(2-1)!} * \frac{4!}{2!(4-2)!} =$ 12 p(T=25) = 12 / 20 = 0.60

$N_{30} = \frac{4!}{3!(4-3)!} =$ 4 p(T=30) = 0 / 20 = 0.20

17 The time to failure in hours of an important piece of electronic equipment used in the manufacture of a DVD player has the probability density function:

Extra problem

$$f(x) = \begin{cases} \frac{1}{2000}\exp^{\frac{-x}{2000}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

a) Find F(x) = $F(t<x) = P(-\infty < t < x) = \int_{-\infty}^{0} 0\,dt + \int_{0}^{x} \frac{1}{2000}\exp^{\frac{-t}{2000}}\,dt$

$$= -\exp^{\frac{-t}{2000}}\Big|_{0}^{x} \quad \text{for } x > 0$$

$$0 \qquad \text{elsewhere}$$

$$= -\exp^{\frac{-x}{2000}} + 1 \quad \text{for } x > 0$$

$$0 \qquad \text{elsewhere}$$

b) Determine the probability that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.

Extra problem

$$P(x > 1000) = 1 - P(x < 1000)$$

$$= 1 - \left\{ -\exp^{\frac{-x}{2000}} + 1 \right\} \quad \text{for } x > 0$$

$$= \exp^{\frac{-1000}{2000}} = 0.61$$

0.61

c) Determine the probability that the component fails before 2000 hours.

Extra problem

$$P(X<2000) = 1 - \exp\left\{\frac{-2000}{2000}\right\}$$

$$= 1 - 0.3679$$

$$= 0.63$$

18 Seven TVs include 2 defectives. Will purchase 3.
If x=defective sets purchased,

$$Nbad = \frac{2!}{x!(2-x)!} \qquad Ngood = \frac{5!}{(3-x)!(2+x)!} \qquad Ntotal = \frac{7!}{3!(7-3)!} = 35$$

Extra

a) Make a table showing all possible values of x and f(x)

$$Pr = \frac{Ngood*Nbad}{Ntotal}$$

$$f(x=0) = \frac{\frac{2}{1(2)} * \frac{5*4*3!}{3!(2)}}{35} = \frac{10}{35} = 2/7$$

$$f(x=1) = \frac{\frac{2}{1(1)} * \frac{5*4*3*2!}{2!(3*2)}}{35} = \frac{20}{35} = 4/7$$

$$f(x=2) = \frac{\frac{2!}{2!} * \frac{5*4!}{1!(4!)}}{35} = \frac{5}{35} = 1/7$$

x	0	1	2
f(x)	2/7	4/7	1/7
F(x)	2/7	6/7	7/7

b) Do histogram

Extra